MATHEMATICS 9

Operations and Number Concepts

Distance Learning

Module 1

Alberta
EDUCATION

Mathematics 9

Module 1

OPERATIONS AND NUMBER CONCEPTS

Distance Learning

Alberta EDUCATION

Mathematics 9
Student Module Booklet
Module 1
Operations and Number Concepts
Learning Technologies Branch
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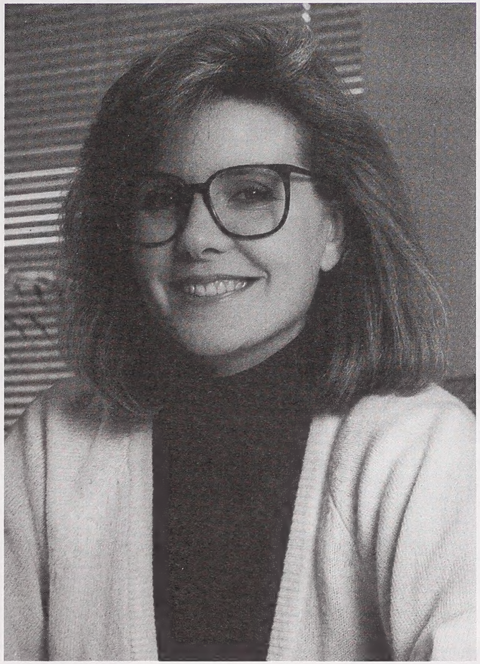
This document is intended for	
Students	✓
Teachers (Mathematics 9)	✓
Administrators	
Parents	
General Public	
Other	

The Learning Technologies Branch has an Internet site that you may find useful. The address is
http://ednet.edc.gov.ab.ca/level3/ltb/300/ltbhome.html.

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

Welcome



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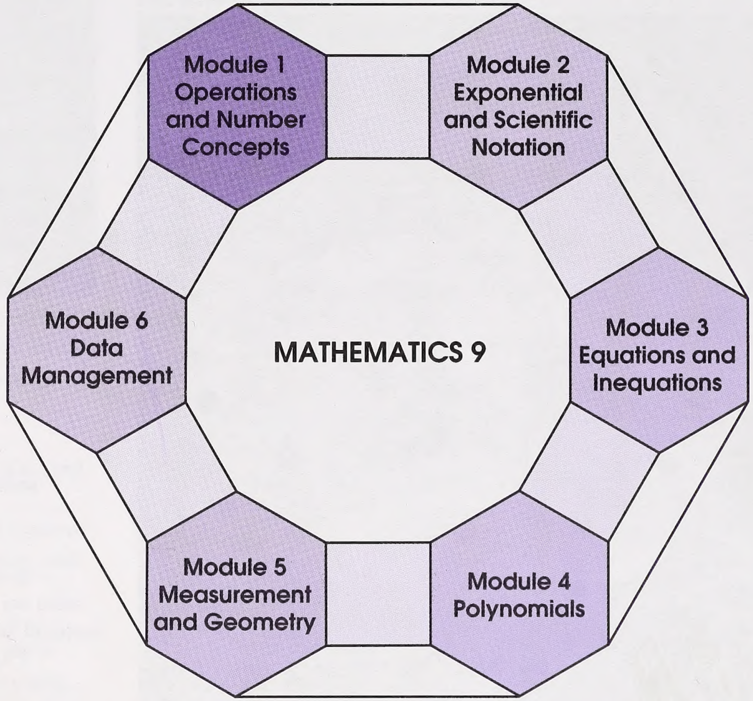
Welcome to Module 1. We hope you'll enjoy your study of Operations and Number Concepts.

Mathematics 9 contains six modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.



The document you are presently reading is called a Student Module Booklet. You may find visual cues or icons throughout it. Read the following explanations to discover what each icon prompts you to do.

- Prepare for a problem that will provide a change of topic.

- Prepare for a challenging problem related to the topic of the activity.

- Use the Internet to explore a topic.

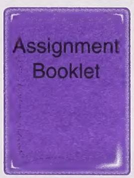
- Use computer software.

- Use a scientific calculator.

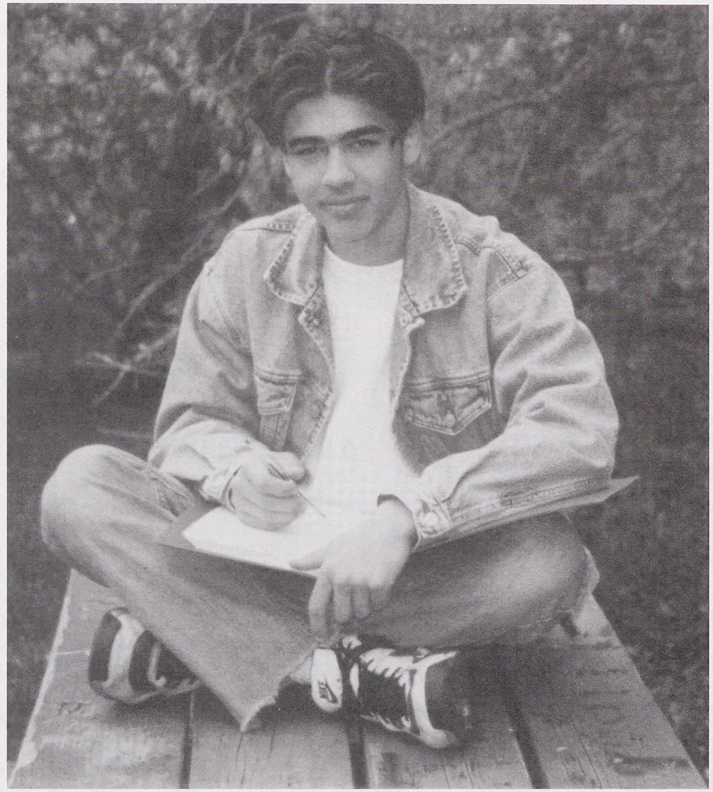
- View a videocassette.

- Pay close attention to important words or ideas.

- Use the suggested answers in the Appendix to correct activities.



- Answer the questions in the Assignment Booklet.



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There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

Technology

Today society is turning to **technology** more than ever before, and it is to your advantage to be able to effectively use technology when required.

Technology is the application of tools, materials, and processes to the solution of problems. More specifically, technology refers to devices and systems that are used in processing, transferring, storing, and communicating information through electronic media.

In Mathematics 9, along with the course materials, you will use a calculator, computer, and videocassette player as tools for learning and doing mathematics.

Calculators are helpful tools for solving problems and exploring patterns and relationships between numbers. Using a calculator will also save you time and help you develop your estimating skills. Therefore, you will be given numerous opportunities in each module to use a calculator.

Computers are useful for organizing and displaying data, or drawing figures. For this reason you will have the chance in many activities to work with popular computer applications such as spreadsheets and draw programs. You will also want to check out the many Internet connections in each module.

Videocassette players allow you to view video programs on key concepts that are difficult to explain in print. That is why video programs are cited in this course.

It is expected that all students will be able to view the video programs and use a calculator, and that most students will do the computer activities. However, if you are unable to access a computer, you may do the calculations using a calculator or draw figures and graphs by hand.

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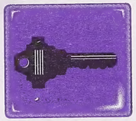
Problem-Solving Skills

One of the exciting features of this course is that you will develop and improve your ability in problem solving. You will need these problem-solving skills many times in your lifetime. Since this course focuses on problem solving, it is important that you understand what a **problem** is.



A problem is a task for which the method of finding the answer (as well as the answer) is not immediately known.

Like any skill, the skill of problem solving must be developed. Problems may or may not involve computation (adding, subtracting, multiplying, and dividing). Some problems are realistic; others are puzzles.

You will have the opportunity in most activities to try a problem-solving challenge.

The  icon is a cue that the problem will be related to the topic of the activity.

The  icon is a cue that the problem will provide a change of topic.

The Four-stage Process

There are four stages that can be used to solve any problem: understanding the problem, developing a plan, trying the plan, and looking back.

Understanding the Problem

In this stage you should expect to feel puzzled. There are various reasons for feeling this way.

- You may not know the meanings of all the words.
- You may not understand the situation in the problem.
- You may be confused by unnecessary information.

Once you understand the problem, you should think about the problem and make an estimate of what the answer should be. This will help you arrive at a reasonable answer.

Developing a Plan

This is where you should decide on the plan of action that you are going to take to solve the problem. You may consider the following strategies:

- changing your point of view
- using objects
- using diagrams
- making an organized list
- using Venn diagrams
- making a table
- guessing, checking, and revising
- acting out a problem
- working backwards
- simplifying a problem
- finding and applying a pattern
- using elimination
- using truth tables
- using an equation

Note: The Appendix in this module explains these strategies in detail. When you see a problem-solving icon in any module, you should turn to this Appendix and review the problem-solving strategies.

Trying the Plan

In this stage you should try the plan and see if it works.

Be sure to work carefully and record your progress. You are encouraged to use a calculator to help with your calculations.

Note: While trying the plan, you should monitor your progress in order to determine if your plan will lead to a solution. You may find that the plan will not produce a solution, in which case a new plan will have to be developed.

Looking Back

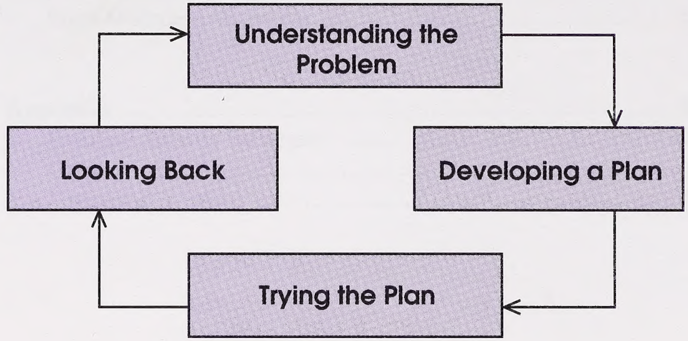
In this stage you should look back at the problem and compare your answer to the estimate you made in the first stage. Restate the problem using your answer.

Ask yourself these questions: "Did my plan work? Is my answer reasonable?"

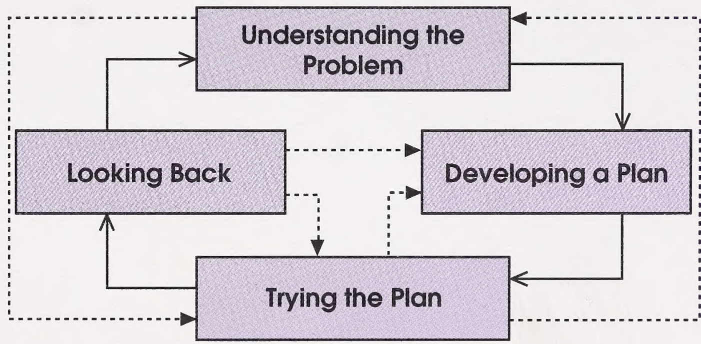
If you did not arrive at an answer, another strategy may work better. If your answer is unreasonable, you may have made errors while trying your plan.

Sequence of Stages

You usually approach a problem in the order outlined in the following diagram.



If you encounter difficulties in your original plan, or if you realize that another strategy will have better results, you may need to return to an earlier stage or use the stages in a different sequence.



CONTENTS

Module Overview

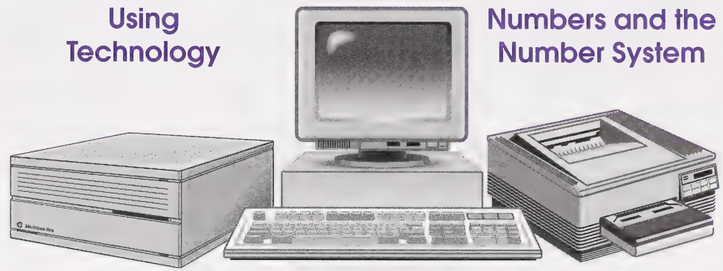
Can you imagine a world without calculators or computers? Computers and calculators are rapidly becoming very common in student learning. For instance, calculators are now used to perform complex arithmetic operations with all types of numbers. Calculators and computers are used by people such as stock brokers, accountants, scientists, engineers, store clerks, owners of small businesses, and people in home businesses, just to name a few. Indeed, calculators and computers are popular in all types of occupations.

In this module you will use a scientific calculator to perform arithmetic operations with rational numbers. You will discover which operations need to be done first in order to produce the correct answer and which operations have to be done first in order to achieve the most efficient results. You will also see what kinds of numbers make up the rational number system, and you will discover more about each of these kinds of numbers.

Module 1
Operations and Number Concepts

Section 1
Using Technology

Section 2
Numbers and the Number System

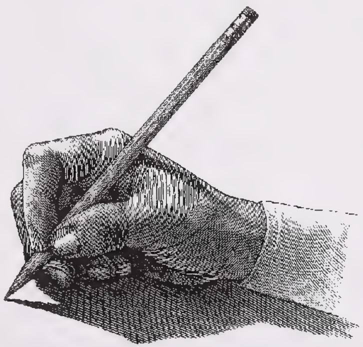


Evaluation

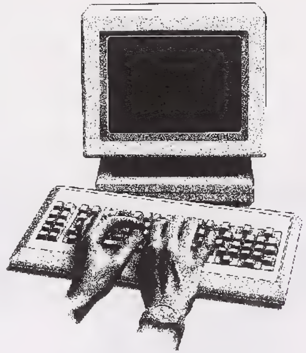
Your mark for this module will be determined by how well you complete the assignments at the end of each section and at the end of the module. In this module you must complete two section assignments and a final module assignment. The mark distribution is as follows:

Section 1 Assignment	**40 marks**
Section 2 Assignment	**20 marks**
Final Module Assignment	**40 marks**
TOTAL	**100 marks**

When doing the assignments, work slowly and carefully. You must do each assignment independently, but if you are having difficulties, you may review the appropriate section in this module booklet.



If you are working on a CML terminal, you will have a module test as well as a module assignment.



Note

There is a supervised final test at the end of this course. Your mark for the course will be determined by how well you do on the module assignments and the supervised final test.

Section 1: Using Technology

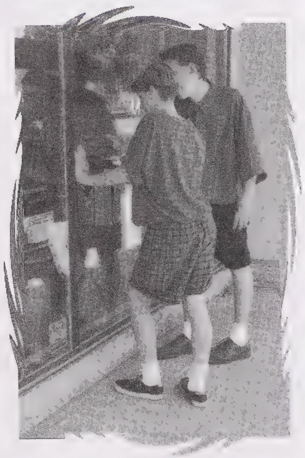
Stock market analysts use computers to track the vast array of stocks on the different stock exchanges. They use calculators and computers to quickly total the cost of several stock purchases for a customer, thus making calculators and computers an integral part of the business world.

Do you use a calculator or a computer spreadsheet program to help you answer mathematical problems? Can you use a calculator effectively and efficiently?

In this section you will use a calculator to perform single-operation and multiple-operation calculations with rational numbers. You will also determine which keystroke sequences are more efficient when solving a particular problem or calculation. If you have access to a computer, you will have the opportunity to apply computer technology to problem solving involving rational numbers.

Activity 1: Solving Problems Involving One Operation

Carla is going back to school next week. In order to be ready, she made a list of the following school supplies with prices she found in a recent flyer.



- loose leaf notebook $2.89
- package of lined paper $1.39
- package of dividers $2.09
- coil-bound notebook $1.69
- package of pens $0.99

Carla has a ten-dollar bill. Does she have enough money?

How would you find the total of the school supplies? You could use a pencil and paper or a calculator to find the answer; but, whichever method you use, you need to be able to determine if your answer is reasonable.

In this activity you will use estimation to approximate the answers to problems involving rational numbers. You will then use a calculator to perform the arithmetic operations and solve these problems.

To find the total of Carla's list, follow these steps.

Step 1: Estimate the sum by rounding to the nearest whole dollar or by using front-end digits.

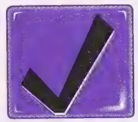
Rounding

$$\begin{array}{r} \$2.89 \\ 1.39 \\ 2.09 \\ 1.69 \\ \underline{+\ 0.99} \end{array} \doteq \begin{array}{r} \$3.00 \\ 1.00 \\ 2.00 \\ 2.00 \\ \underline{+\ 1.00} \\ \$9.00 \end{array}$$

Front-End Digits

$$\begin{array}{r} \$2.89 \\ 1.39 \\ 2.09 \\ 1.69 \\ \underline{+\ 0.99} \end{array} \doteq \begin{array}{r} \$2.00 \\ 1.00 \\ 2.00 \\ 1.00 \\ \underline{+\ 0.00} \\ \$6.00 \end{array}$$

1. Which method of estimation is better in this case? Why?



Check your answers by turning to the Appendix.

Step 2: Calculate the sum using a calculator.

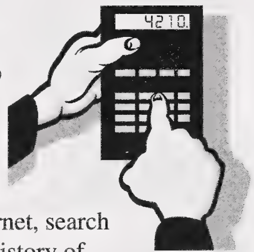
2 • 8 9 + 1 • 3 9 + 2 • 0 9 + 1 • 6 9 + • 9 9 =

9.05

The total cost of the supplies on Carla's list is $9.05.

You have probably been using a calculator to help you solve problems involving decimal numbers for several years. Decimal numbers and fractions are rational numbers, and most examples involving rational numbers in the everyday world are in decimal form.

Did You Know?



The first hand-held electronic calculator was introduced in 1967 by Texas Instruments™. Weighing more than 1 kg, it could handle up to six digits and perform the operations of addition, subtraction, multiplication, and division.

INTERNET

If you have access to the Internet, search for more information on the history of calculators or on calculators in general. Enter the word "calculators" or the words "calculator history" into one of the Internet's search engines, and go from there.

Problems Involving Addition and Subtraction

You will discover that various problem-solving situations (similar to the one in the introduction to Activity 1) involve addition and subtraction. The calculator is a valuable tool in all these situations.

Example 1

An automobile begins a journey from the bottom of Death Valley, 85.9 m below sea level, and climbs 110.9 m. What is the new position of the automobile relative to sea level?

Solution

Estimate by rounding to the nearest integer.

$$\begin{aligned}(-85.9)+(+110.9) &\doteq (-86)+(+111)\\ &\doteq (111-86)\\ &\doteq 25\end{aligned}$$

Since the problem reads "85.9 m below sea level," this number is negative.

Enter the keystrokes as follows:

8 5 • 9 +/−

+ 1 1 0 • 9 =

25.

Therefore, the new position of the automobile is 25 m above sea level.

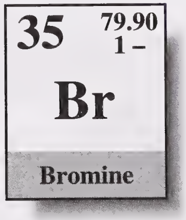
2. Find another way to do this question using fewer keystrokes. How many fewer keystrokes did your method use?

Check your answers by turning to the Appendix.

From your previous years of working with integers, you discovered that subtracting an integer produces the same result as adding the integer's additive inverse. A similar relationship exists between the subtraction and the addition of rational numbers.

Example 2

The boiling point of bromine is 58.8°C. The freezing point of bromine is −7.2°C. What is the difference in temperature between the boiling point and the freezing point of bromine?

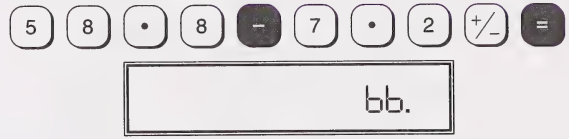


Solution

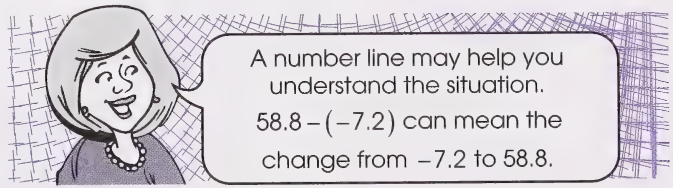
Estimate the answer.

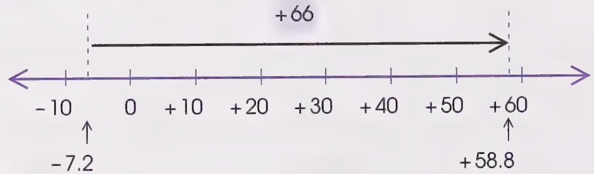
$$\begin{aligned} 58.8-(-7.2) &\doteq 60-(-7) \\ &\doteq 60+7 \\ &\doteq 67 \end{aligned}$$

Enter the keystrokes as follows:



Therefore, the difference between the boiling point and the freezing point of bromine is 66°C.



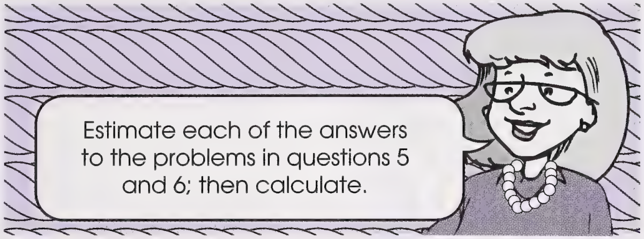


3. Determine a way of finding the answer to Example 2 using fewer keystrokes on a calculator. How many fewer keystrokes does your method use?

4. Find the following sums or differences by estimating and then using a calculator.

a. $-19.8+(-26.4)$ b. $43.7-(-21.4)$

c. $148.5-216.8$ d. $87.6+(-194.2)$



5. At the 1993 World Alpine Ski Championships, Canada's Kate Pace won the gold medal with a time of 1 min 27.38 s. In second place, Astrid Loedemal of Norway finished $\frac{28}{100}$ of a second behind Kate Pace. What was Loedemal's time for the race?

6. Over a five-year period, a company records the following:

- 7.8% loss in 1991
- 2.7% loss in 1992
- 5.6% profit in 1993
- 1.2% loss in 1994
- 11.9% profit in 1995

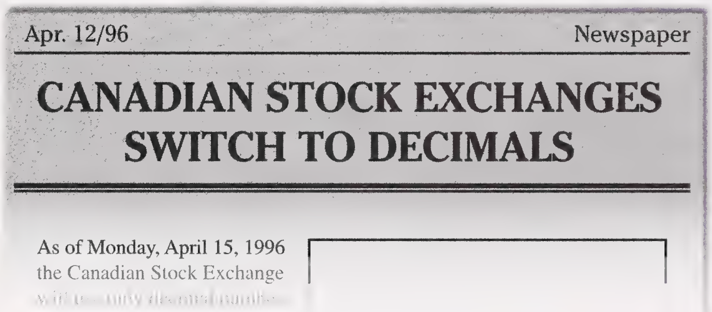
Find the overall profit or loss for the company.

7. Enter the calculator sequence in question 6 in a different way. Does your second sequence use more or less keystrokes?

8. How was the use of the calculator more efficient in question 7?

Check your answers by turning to the Appendix.

One area where addition and subtraction of decimal numbers are used is in the stock market.


Apr. 12/96 Newspaper

CANADIAN STOCK EXCHANGES SWITCH TO DECIMALS

As of Monday, April 15, 1996 the Canadian Stock Exchange

Until April 12, 1996, the Canadian Stock Exchange traded shares valued at $5.00 or more in eights of a dollar. Shares valued below $5.00 were traded in cents. Rational numbers in decimal, fraction, and mixed-number form were used extensively by people involved in the stock market. As of April 15, 1996, with the switch to decimals, stock brokers and investors will only deal with decimal numbers when buying and selling shares on the Canadian stock exchanges.

Newspapers now list each day's transactions on the Canadian stock exchanges in the following manner.

Stock	Volume	High	Low	Close	Change
Shaw Paper	45 000	12.75	12.25	12.50	0.10

The chart shows that 45 000 shares of the stock of Shaw Paper were traded on this particular day. The highest price paid for a share that day was $12.75, and the lowest price was $12.25. At the end of the day, the price being paid for a share was $12.50. This price was up $0.10 from the previous day's closing price.

Example 3

A & A Foods started the trading day at $8.75. At the end of the day, the stock price was down $2.30. What was the trading price at the end of the day?

Solution

Estimate the price.

$$\begin{aligned} 8.75 - 2.30 &\doteq 9 - 2 \\ &\doteq 7 \end{aligned}$$

Enter the keystrokes as follows:

8 . 7 5 − 2 . 3 =

6.45

The trading price at the end of the day was $6.45.

Example 4

Beverly Ltd. opened the trading day at $9.25. At the end of the day the trading price was $7.50. What was the change?

Solution

Estimate the change.

$$\begin{aligned} 9.25 - 7.50 &\doteq 9 - 8 \\ &\doteq 1 \end{aligned}$$

Enter the keystrokes as follows:

9 . 2 5 − 7 . 5 =

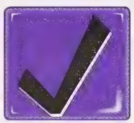
1.75

The change was a decrease of $1.75.

9. Over a one-week period, Telus stock went up $0.15 on Monday, up $0.35 on Tuesday, down $0.25 on Wednesday, down $0.10 on Thursday, and up $0.50 on Friday.

a. If the price per share was $16.25 when the stock market opened on Monday, then what was the closing price at the end of Friday's trading?

b. Is the calculated price close to the estimated price? What does the estimate show you?



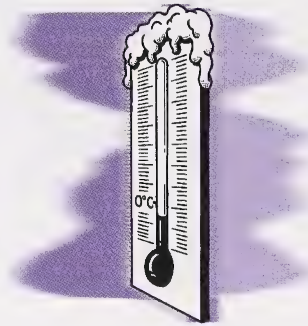
Check your answers by turning to the Appendix.

Problems Involving Multiplication and Division

Many everyday situations require the ability to multiply or divide rational numbers. The most common rational numbers used are decimal numbers. Decimal numbers are often used in money questions, percent discounts, interest problems, and science applications. The calculator will prove even more useful in calculations involving multiplication and division than in those involving addition and subtraction.

Example 1

The present temperature is 0°C. If the temperature has been rising 2.5°C each hour, when was it −10°C?



Solution

Estimate the answer.

$$
\begin{aligned}
(-10)\div(2.5) &\doteq (-10)\div(3)\\
&\doteq -\frac{10}{3}\\
&\doteq -3
\end{aligned}
$$

Since the signs are different, the answer will be negative.

Enter the keystrokes as follows:

1 0 +/_ ÷ 2 • 5 =

-4.

The temperature was $-10°C$ 4 h ago.

Example 2

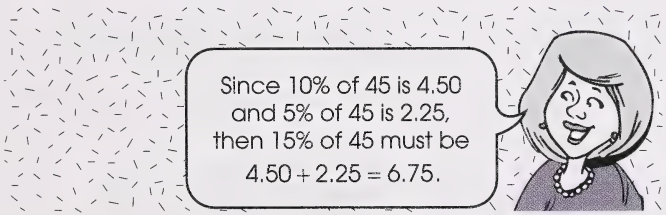
Todd found a shirt and a pair of pants that he wanted in a department store. The price tag on the shirt showed \$16.99, and the price tag on the pants showed \$28.39. When Todd brought these two items to the cashier, the clerk said, "There is 15% off on everything in the store today." How much will Todd save (assuming there is no tax)?

Solution

Estimate.

$$\begin{aligned}16.99 + 28.39 &\doteq 17 + 28 \\ &\doteq 45\end{aligned}$$

$$\begin{aligned}15\% \text{ of } 45 &\doteq 45 \times 0.15 \\ &\doteq 6.75\end{aligned}$$



Enter the following keystrokes.

1 6 • 9 9 + 2 8 • 3 9 =
× • 1 5 =

6.807

Todd will save \$6.81.

10. Find the following products or quotients. Estimate; then use a calculator to find the exact answer.

a. -3.81×5.2 **b.** $84.6 \div 9$

c. $20 \div (-3.5)$ **d.** $-15 \times (-7.1)$

Check your answers by turning to the Appendix.

Sometimes it is necessary to change from fractions to decimals and percents or from percents and decimals to fractions.

It is easy to use a calculator to convert fractions to decimals and percents.

Example 3

Convert $\frac{1}{8}$ to a decimal and to a percent using a calculator.

Solution

1 ÷ 8 =

0.125

× 1 0 0 =

12.5

$\frac{1}{8}$ is equivalent to 0.125 and 12.5%.

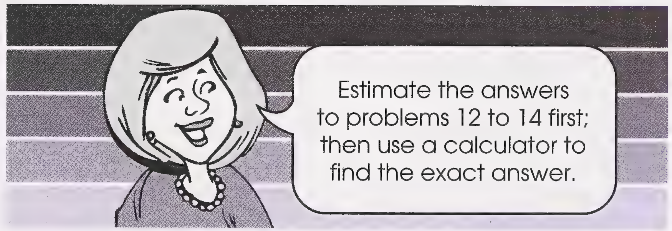
To convert a percent to a decimal, divide by 100. To change a decimal to a fraction, you will need to recall (from your previous studies of mathematics) the common fractions and their decimal equivalents. Here are a few examples.

$\frac{1}{8} = 0.125$	$\frac{1}{6} = 0.1\overline{6}$	$\frac{1}{9} = 0.\overline{1}$
$\frac{3}{8} = 0.375$	$\frac{1}{3} = 0.\overline{3}$	$\frac{2}{9} = 0.\overline{2}$
$\frac{5}{8} = 0.625$	$\frac{5}{6} = 0.8\overline{3}$	$\frac{1}{11} = 0.\overline{09}$
$\frac{7}{8} = 0.875$	$\frac{7}{6} = 1.1\overline{6}$	$\frac{2}{11} = 0.\overline{18}$

These are just a few of the common equivalent fractions and decimals. You can use a calculator to find equivalent fractions by trial and error.

11. To review changing fractions to decimals and percents and vice versa, copy and complete this chart in your notebook.

Fraction	Decimal	Percent
		37.5%
	$0.1\overline{6}$	
$\frac{7}{8}$		
	1.25	
		$83\frac{1}{3}\%$
$\frac{5}{9}$		
		110%



12. The Hoover Dam on the Colorado River in the United States contains 2 500 000 m^3 of concrete. At 8.5 m^3 per truckload, approximately how many truckloads of concrete did it take to complete the dam?

13. Mr. Gervais and his son Stan bought a computer, a monitor, and a printer for their home. Although the computer package is regularly priced at \$2295, they got it at a special back-to-school sale for 15% off.

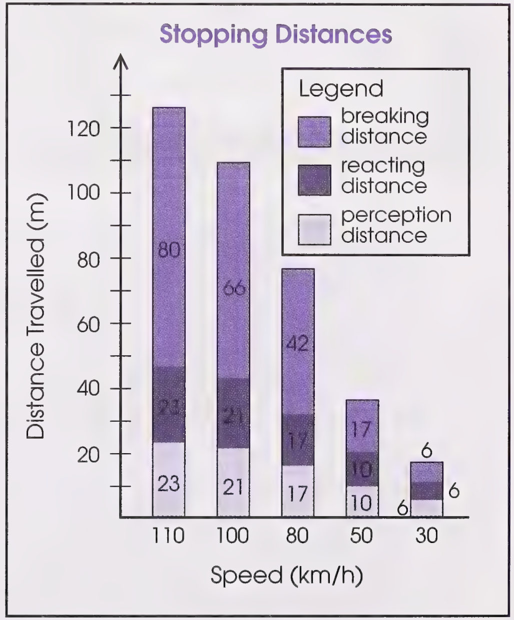
a. How much did they save?

b. What was the sale price?

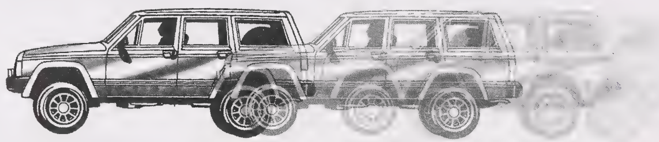
c. How much did they pay including 7% GST?

Use the following information to answer question 14.

When driving, the time it takes for your foot to reach the brake pedal is called reaction time. The braking distance is the distance travelled after your brakes are applied. A third factor that affects stopping time and distance is perception time. This is the time it takes for a driver to see a situation and realize that the vehicle must be stopped. The average perception time is about $\frac{3}{4}$ s. Of course, all of these factors are also influenced by road conditions, the condition of the car, and the condition and experience of the driver. The graph shows some stopping distances under normal road conditions.



SOURCE: DATA FROM ALBERTA REGISTRIES



14. Suppose a person driving a car at 100 km/h has a perception time of 1 s. **Remember:** The graph is based on an average reaction time of $\frac{3}{4}$ s.

a. How far would the car travel before the driver, seeing a dangerous situation, realizes the vehicle must be stopped?

b. Calculate the total distance the car would travel from the time the driver first sees a dangerous situation until the vehicle is stopped.

c. The length of an average car is 6 m. How many car lengths is the distance calculated in question 14.b.?

d. When following another vehicle, the rule is to leave at least one car length (or 6 m) for every 15 km/h. How many car lengths should the driver stay behind a vehicle travelling at the same speed (100 km/h)?

e. If the stopping distance at 100 km/h is equivalent to 19 car lengths, how can the driver safely remain 7 car lengths behind while travelling at 100 km/h? Explain.

Use a computer and a spreadsheet program, such as *Clarisworks*™, to answer question 15.

If you do not have access to a computer and a spreadsheet program, set up a chart and use a calculator to answer the question.

15. Obtain advertising flyers from several stores, and find several items that are common to all the stores. You could choose various grocery stores, hardware stores, drugstores, or department stores. Set up and complete a chart similar to the following. Determine which store gives you the best value for your money if you buy all the items at one store.

	A	B	C	D
1	**Item**	**Store 1 Cost**	**Store 2 Cost**	**Store 3 Cost**
2				
3	peas			
4	chicken soup			
5	hamburger			
6				
7	**Total**			

If you are using *Clarisworks*™ or a similar spreadsheet program, you need to enter your information in each of the rectangular spaces (or cells). You can enter words, numbers, or formulas in each cell. The cells are labelled A1, A2, A3, …; B1, B2, B3, …; and so on. In the cells across from Total (cells B7, C7, D7), you need to enter a formula which will automatically add up the cost of the items for a particular store. For example, in cell B7, enter the formula =B3+B4+B5. This formula will give you the sum of cells B3, B4, and B5.

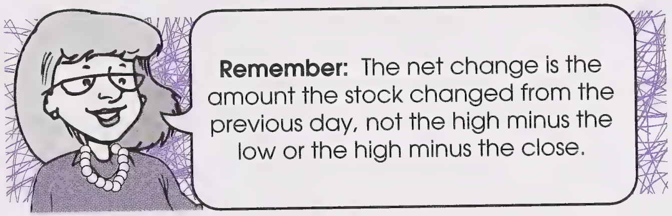
If you need additional help with using a spreadsheet program (such as opening the spreadsheet program or starting a new spreadsheet), consult the user's guide for the program.

Use the following chart for question 16.

Stock	Vol.	High	Low	Close	Net Change
High Oil	956 204	12.40	12.00	12.25	+0.10

16. Ria owns 1200 shares of High Oil. The stock performance for one day is listed in the previous chart. Determine the following:

a. the change in the value of each share
b. the change in the value of all the High Oil shares Ria owns
c. the total value of Ria's shares at the end of the day



Use the following information to answer questions 17 to 19.

A Boeing 757 aircraft departs from Edmonton at 11:20 P.M. and arrives in Montreal 4.5 h later. The distance is about 2976 km. The plane's cruising speed is about 830 km/h. The Boeing 757 has a fuel capacity of 34 200 kg; it consumes about 90 kg of fuel per minute in flight.

17. What time does the plane land in Montreal? (**Hint:** When it is 8:00 A.M. in Edmonton it is 10:00 A.M. in Montreal.)

18. a. A formula to calculate average speed is $s = \frac{d}{t}$, where s is average speed (in kilometres per hour), d is distance (in kilometres), and t is time (in hours). Calculate (to the nearest kilometre per hour) the average speed of the plane during the flight from Edmonton to Montreal.

b. How much greater than or lesser than the cruising speed is the average speed? Can you think of a reason for this difference? Explain.

19. a. About how much fuel is consumed during the flight to Montreal? Assume no wind.

b. If the plane were to continue on to Miami, Florida, would the plane have to be refuelled for the flight from Montreal to Miami (a distance of 1413 km)?

20. Brandon was vacationing in the United States, where temperatures are given in degrees Fahrenheit (°F). To convert the temperature from degrees Fahrenheit (°F) to degrees Celsius (°C), he used the following formula.

$$^{\circ}\mathrm{C} = \frac{5}{9}(^{\circ}\mathrm{F} - 32)$$

a. If the temperature was 93°F, what was the equivalent temperature in degrees Celsius?

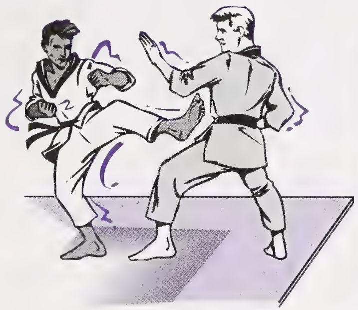
b. Write out the keystrokes needed to answer question 20.a. using a calculator. Write out two different keystroke sequences.

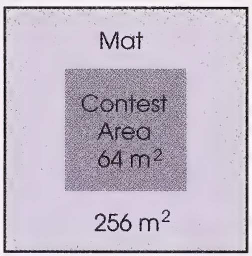
21. Hamel and Annette work as lifeguards at a swimming pool. Hamel earns \$7.00 per hour and Annette, as a pool supervisor, earns \$7.50 per hour. Hamel and Annette work the following hours for one particular week during the summer. Calculate the amount of money each earns during that week.

Name	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Hamel	—	4.5	5	2.5	5.5	7.5	8.5
Annette	5.5	3	6.5	3.5	4.5	7.5	—

Check your answers by turning to the Appendix.

Square Roots of Numbers





In a wrestling or judo match, part of the objective is to remain within certain bounds of the mat.

In international judo, the contest area is a square covering $64\ m^2$ on a mat that is $256\ m^2$. What is the distance from the outer edge of the contest area to the edge of the mat? To answer this question you need to be able to determine the square root of a number.

In previous mathematics courses, you studied the square root of a perfect square and of any number using a calculator. In this course, continue using a calculator to find square roots.

Example 1

What number squared is equal to 64?

Solution

Let n be the number.

$$n^2 = 64$$
$$n = \text{the square root of } 64$$
$$n = 8 \text{ or } -8$$

Either number squared equals 64.

The square root of 64 can be either 8 or −8 since $8 \times 8 = 64$ and $(-8)\times(-8) = 64$.

Example 2

If a square has an area of $36\ \text{cm}^2$, what is the measure of one side?

Solution

Use the formula $A = s^2$.

$$A = s^2$$
$$36 = s^2$$
$$s = \sqrt{36} \quad \leftarrow \text{square root of 36}$$
$$s = 6 \text{ or } -6$$

The measure of one side is 6 cm.

22. Do you agree that the measure of the side should be 6 cm and not -6 cm? Why?

Check your answers by turning to the Appendix.

When you take the square root of a number like 36, the square root is 6 or -6 since $6 \times 6 = 36$ and $(-6) \times (-6) = 36$. However, if you want to find the length of a side or a distance, then use only the positive square root.

Another name for the positive square root is the **principal square root**. The symbol $\sqrt{\ }$, called a radical symbol, is used to represent the principal square root.

To avoid confusion, mathematicians use the radical symbol to represent positive square roots. If the negative square root is desired, then a negative symbol is used in front of the radical sign.

Thus, $\sqrt{36} = 6$ and $\sqrt{6.4} = 0.8$, while $-\sqrt{36} = -6$ and $-\sqrt{6.4} = -0.8$.

23. Evaluate the following:

a. $\sqrt{100}$ **b.** $\sqrt{256}$ **c.** $-\sqrt{81}$ **d.** $-\sqrt{400}$

Check your answers by turning to the Appendix.

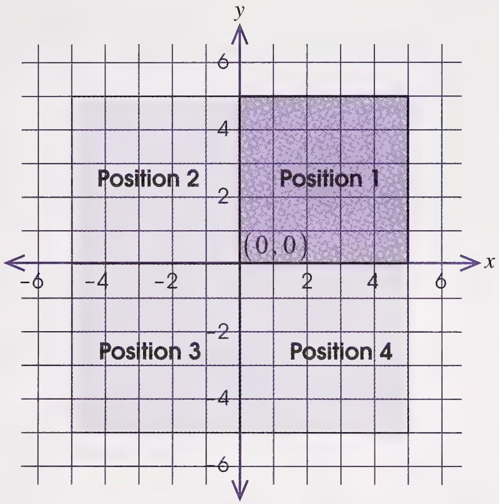
Sometimes it is appropriate to use the principal square root only, and sometimes it is appropriate to use the positive and negative square roots. You have seen an example involving area (Example 2), where only the principal square root is used. The following example shows where both the positive and negative square roots are appropriate.

Example 3

A coordinate plane has a square with an area of 25 square units with one corner at coordinate $(0,0)$. Find the location of all the other possible vertices.

Solution

The square could be in any one of the four locations. To find the length of a side, take the principal square root of 25 to get 5. However, to place the vertices you need both positive and negative 5.



The other three vertices for each position are as follows:

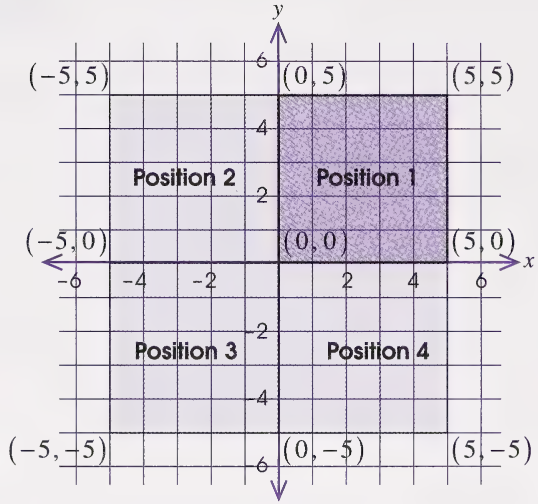
Position 1: $(5,0)$, $(5,5)$, and $(0,5)$

Position 2: $(0,5)$, $(-5,5)$, and $(-5,0)$

Position 3: $(-5,0)$, $(-5,-5)$, and $(0,-5)$

Position 4: $(0,-5)$, $(5,-5)$, and $(5,0)$

These vertices are shown on the following graph.



Use a calculator (where necessary) to answer the following questions.

24. A circle has an area of $153.7\ \text{cm}^2$. What is the radius of the circle to the nearest centimetre? $\left(\textbf{Hint:}\ A = \pi r^2\right)$ Estimate your answer first.

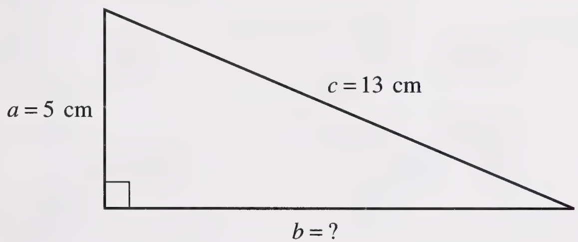
25. **a.** Why did you only use the positive value of the radius in question 24?

b. What name is given to this value?

c. Can a calculator give a negative value for the square root of a positive number?

d. Try to take the square root of -49. What happens? Why?

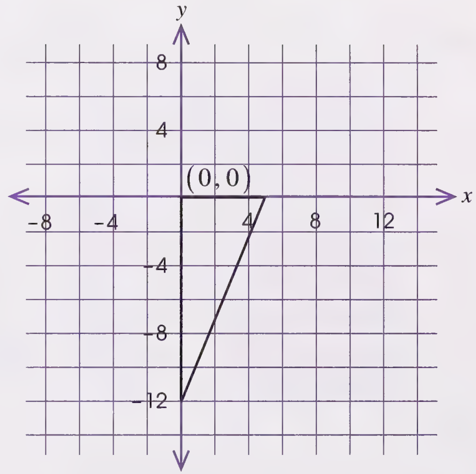
26. You are given the dimensions of two sides of a right triangle in the following diagram.



a. State the relationship that you would use to find side b.

b. Calculate the length of side b. Do you use both the positive and negative square roots or the positive (principal) square root only? Explain.

c. If the triangle is on a coordinate plane with the right angle at $(0, 0)$ (as shown in the following diagram), what are the coordinates of the other two vertices?



d. Did you use both positive and negative numbers to name the coordinates? Are these numbers lengths?

Check your answers by turning to the Appendix.

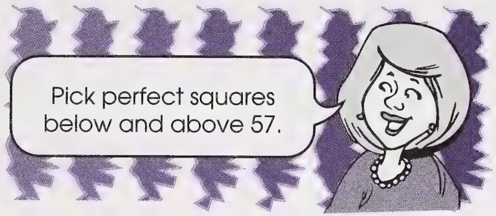
You have used a calculator to find the square root of a number in previous mathematics courses. A calculator is a valuable tool for finding the square root of non-perfect squares. Whenever you use a calculator to find the square root of a number, estimate first.

Example 4

Estimate $\sqrt{57}$; then calculate using a calculator.

Solution

Estimate



$$49 < 57 < 64$$
$$\sqrt{49} < \sqrt{57} < \sqrt{64}$$
$$7 < \sqrt{57} < 8$$

Thus, $\sqrt{57} \doteq 7.5$, since 57 is about half way between 49 and 64.

$$\underbrace{49 < 57}_{8} \underbrace{< 64}_{7}$$

Calculator

√ 5 7 =

7.549834435

Use a scientific calculator to answer question 27.

27. Estimate the square root of the following; then calculate to two decimal places.

a. $\sqrt{20}$	**b.** $\sqrt{7}$	**c.** $\sqrt{1215}$
d. $-\sqrt{32}$	**e.** $\sqrt{15}$	**f.** $\sqrt{873}$

28. Use the information from the answers to question 27 to answer the following questions.

a. What do you notice about the number of digits in each calculator display?

b. If the calculator display showed more digits, would there be more digits in each display?

c. What are the numbers in the answers called?

Check your answers by turning to the Appendix.

In general, decimal numbers that do not repeat or terminate are known as irrational numbers. All square roots of non-perfect squares are irrational numbers. Examples of irrational numbers are $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, and $\sqrt{8}$. You may study more about irrational numbers in a future mathematics course.

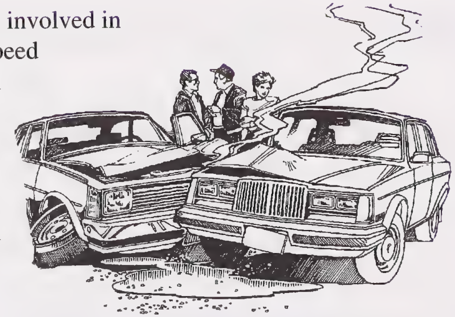
29. Catherine has a square cross-stitch picture with sides measuring 24 cm. She wants to mount it on a piece of drymount board that has twice the area of the cross-stitch picture. Calculate the length of each side of the required piece of drymount board to the nearest centimetre. Estimate the length first.

30.

Jerrold and Mabel own a mixed farm. They want to fence a 4 ha, square area for a sheep pasture. How many metres of fencing are required for the pasture? ($1\ \text{ha} = 10\,000\ \text{m}^2$)

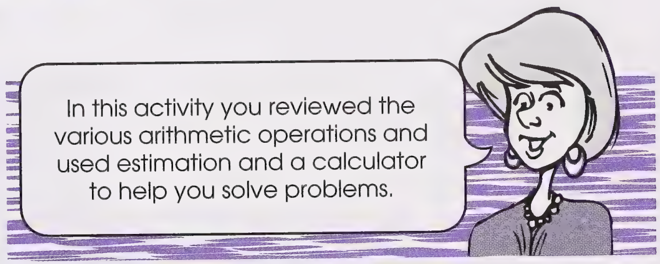
The formula $s = \sqrt{150b}$, where s is speed (in kilometres per hour) and b is braking distance (in metres), can be used to calculate the speed of a motor vehicle given the braking distance. Use this information to answer question 31.

31. Gina and Karl were involved in an accident. The speed limit in the location of the accident was 50 km/h. The police officer, who arrived after the collision, measured the skid marks and gave Gina a ticket. If the skid marks were 28 m in length, then explain why the police officer gave Gina the ticket.



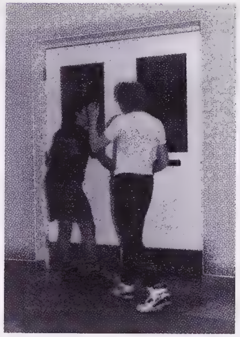
32. Suppose you know the area of a square. Explain, in words, how you would determine the perimeter of the square.

Check your answers by turning to the Appendix.



Activity 2: Solving Problems Involving Multiple Operations

Read over the following list of activities.



- Get dressed.
- Have breakfast.
- Shower.
- Go to class.
- Get out of bed.
- Catch the bus for school.
- Brush your teeth.

In what order would you put these activities? Why is order so important in this case?

In this activity you will solve rational number problems involving more than one operation with a calculator. You will also discover why it is important to perform arithmetic operations in a particular order.

Suppose you have the following calculation.

$$(24.75 + 18.98 - 1.49) \times 1.07$$

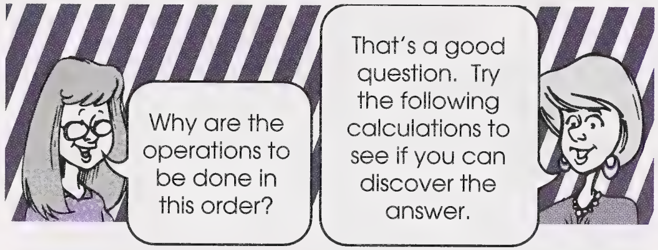
This question involves the operations of addition, subtraction, and multiplication. How do you know which operation to do first?

Order of Operations

Do you recall the acronym BEDMAS? The letters stand for various operations and are to be done in the order given.

B—**b**rackets
E—**e**xponents
D—**d**ivision or
M—**m**ultiplication
A—**a**ddition or
S—**s**ubtraction

1. Write out, in words, how the order of operations are to be done according to BEDMAS.



2. **a.** $0.50 - 0.70 \times 0.30$ **b.** $(0.50 - 0.70) \times 0.30$

3. In question 2.a., which operation did the calculator do first?

4. What effect did the brackets have on the calculator in question 2.b.?

5. Are the answers to questions 2.a. and 2.b. the same? Explain.

Check your answers by turning to the Appendix.

Now study the following examples which illustrate the various operations performed with a calculator.

Example 1

Calculate $(24.75+18.98-1.49)\times 1.07$.

Solution

Estimate

$$\begin{aligned}(24.75+18.98-1.49)\times 1.07 &\doteq (25+19-1)\times 1 && \leftarrow \text{Perform the operations in the brackets.}\\ &\doteq (43)\times 1 && \leftarrow \text{Multiply.}\\ &\doteq 43\end{aligned}$$

Calculator

(2 4 • 7 5 + 1 8 • 9 8
− 1 • 4 9) × 1 • 0 7 =

45.1968

Example 2

Calculate $4+(-12)\div(-15)$.

Solution

Estimate

$$\begin{aligned}4+(-12)\div(-15) &= 4+\left(\frac{-12}{-15}\right) && \leftarrow \text{Divide.}\\ &\doteq 4+1 && \leftarrow \text{Add.}\\ &\doteq 5\end{aligned}$$

Calculator

4 + 1 2 +/− ÷ 1 5 +/− =

4.8

Notice that brackets are not required to have the calculator do multiplication before addition or to have a negative sign attached to a number. Most calculators automatically do multiplication and division before addition and subtraction.

Perform the following calculations using a calculator. Follow the rules associated with the acronym BEDMAS. Be sure to estimate before you use your calculator.

6. **a.** $5.27 + 9.3 \times 2.1$ **b.** $-3.98 \times 1.4 \div 0.7 + 7.56$

c. $(3.34 - 7.21)4 - 3.50$ **d.** $\dfrac{5.3 + 25.1 - 19.3}{4}$

7. Why were the brackets used in the calculator answer to question 6.d.?

8. In question 6.d., what other keystroke could you use in place of the brackets in order to obtain the same answer? Is this a better way to perform the calculations? Explain.

9. Estimate the total cost for the following before taxes.

a. 2 CD's @ $11.99 each and 1 audiotape @ $7.99

b. 3 pairs of socks @ $2.89/pair and 2 pairs of jeans @ $34.98/pair

c. 6 cans of peas @ 3 for $1.89

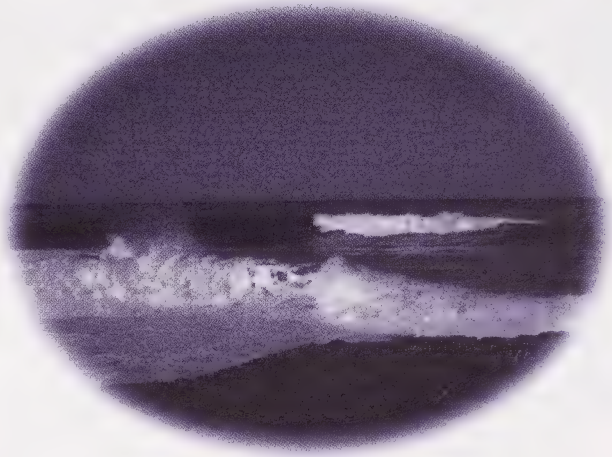
Check your answers by turning to the Appendix.

A calculator can be used to help solve problems involving several operations that must be performed in a specified order. The following examples illustrate order of operations with the use of a calculator in problem solving.

Example 3

Have you ever looked out over the ocean? The distance you can see out over the ocean, when standing on the shore, is given by the formula $d = 3.6\sqrt{h}$, where d is the distance (in kilometres) and h is the height of your eyes above the water (in metres).

Find the distance you can see if your eyes are 2.5 m above the water. Round your answer to the nearest tenth.



Solution

Substitute into the formula.

$$\begin{aligned} d &= 3.6\sqrt{h} \\ &= 3.6\sqrt{2.5} \end{aligned}$$

Enter the following keystrokes.

3 • 6

× √ 2 • 5 =

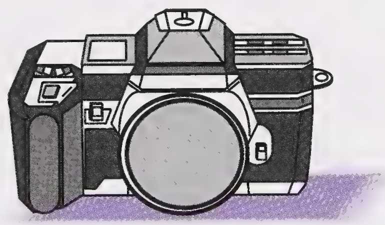
5.692099788

The calculator finds the square root first; then it multiplies.

Therefore, you can see approximately 5.7 km out over the ocean.

Example 4

A department store had cameras on sale for 15% off the regular price. One type of camera was still not selling well; so it was discounted a further 10%. Find the sale price of this camera if it was regularly priced at $425. Find the total cost after 7% GST is added.



Solution

Estimate the sales price.

$$\begin{aligned} &[425-(425\times 15\%)]-\{[425-(425\times 15\%)]\times 10\%\} \\ &\quad =[400-(400\times 0.15)]-\{[400-(400\times 0.15)]\times 0.10\} \\ &\quad \doteq (400-60)-(400-60)\times 0.10 \\ &\quad \doteq 340-(340\times 0.10) \\ &\quad \doteq 340-34 \\ &\quad \doteq 306 \end{aligned}$$

The sale price is approximately $306.

Estimate the sale price including GST.

$$\begin{aligned} 306+306\times 7\% &\doteq 300+300\times 0.07 \\ &\doteq 300+21 \\ &\doteq 321 \end{aligned}$$

The sale price including GST is approximately $321.

Enter the following keystrokes to determine the sale price.

4 2 5 − 4 2 5

× . 1 5 −

36 1.25

15% = 0.15

(4 2 5

− 4 2 5

× . 1 5)

36 1.25

× . 1 =

325. 125

Notice only one set of brackets was required to do this calculation on a scientific calculator.

The camera is on sale for \$325.13.

Find the total cost including GST.

$$325.13 + 325.13 \times 0.07$$

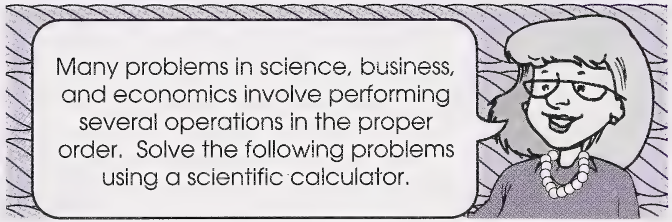
Enter the following keystrokes.

3 2 5 . 1 3

+ 3 2 5 . 1 3 × . 0 7 =

347.889 1

The total cost of the camera is \$347.89.



10. Samantha bought a new bicycle that cost \$375 before tax. If the GST is 7% and the PST is 5%, how much did Samantha have to pay altogether?

11. The Van Dusen family purchased a new house in Edmonton for \$135 000. Two years later, they sold it to the O'Rourkes for 5% more than they paid for it (not including GST). Six months later, Mr. O'Rourke got transferred and they ended up selling the house for 5% less than their purchase price. Ignore all additional charges such as legal fees and document fees.

a. How much did the Van Dusen family pay including GST?

b. How much did the O'Rourkes pay for the house?
Note: GST is paid on new houses only.

c. What did the O'Rourkes sell the house for?

12. Matthew is collecting quarters and dimes. The face value of his collection is given by the following equation.

$$v = 0.25q + 0.1d$$

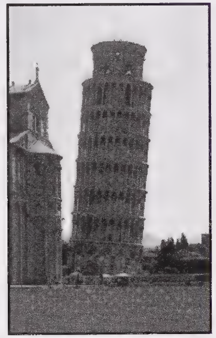
What is the face value of his collection if he has 31 quarters and 82 dimes?

The formula for the time it takes any object to fall from a given height is $t = \sqrt{\frac{h}{4.9}}$ where t is time (in seconds) and h is height (in metres). Use this formula to answer questions 13 to 15.

13. The distance from the top of the High Level Bridge in Edmonton to the riverbed is 41.8 m. If the North Saskatchewan River is presently flowing at a depth of 2.1 m, how long would it take a stone to reach the water if it were dropped from the top of the bridge? (Round your answer to one decimal place.)

14. The apex of the largest pyramid at the Muttart Conservatory in Edmonton is 24.1 m above the ground. Suppose a drop of water inside this pyramid falls from the apex directly to the ground. How long would it take the drop of water to reach the ground? (Round your answer to one decimal place.)

15. It was Galileo who first showed that the mass of an object does not affect the speed at which it falls. It is believed he did this by dropping two objects of different masses from the Leaning Tower of Pisa in Italy. The two objects hit the ground at the same time. How long did it take for the two objects to reach the ground if the distance the objects fell was 54.5 m? (Round your answer to one decimal place.)


THEO VAN BOXEL

Check your answers by turning to the Appendix.

You may wish to look for information about the Leaning Tower of Pisa on the Internet. Type in the words “leaning tower of pisa” on any of the search engines on the Internet. You may find a wealth of information as well as a number of photographs.

Now Try This

Use one of the problem-solving strategies to solve the following:

Use a calculator to answer question 16.

16. Find each of the following square roots.

a. $\sqrt{4}$ **b.** $\sqrt{40}$

c. $\sqrt{400}$ **d.** $\sqrt{4000}$

e. $\sqrt{40\,000}$ **f.** $\sqrt{400\,000}$

17. Look at your answers to question 16. Can you see a pattern? Explain.

18. Use the conclusions you reached in question 17 to help you answer the following. **Do not** do any calculations.

a. If $\sqrt{16} = 4$, then $\sqrt{160\,000} =$ ____.

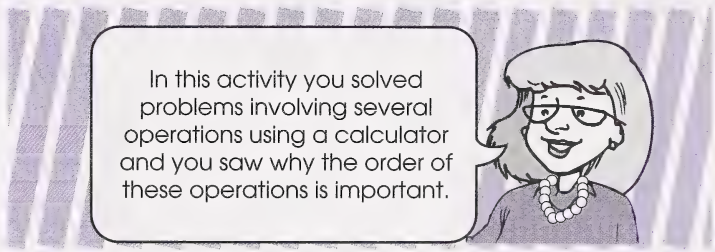
b. If $\sqrt{39} \doteq 6.245$, then $\sqrt{3900} \doteq$ ____.

c. If $\sqrt{225} = 15$, then $\sqrt{2.25} =$ ____.

d. If $\sqrt{7.5} \doteq 2.7386$, then $\sqrt{750} \doteq$ ____.

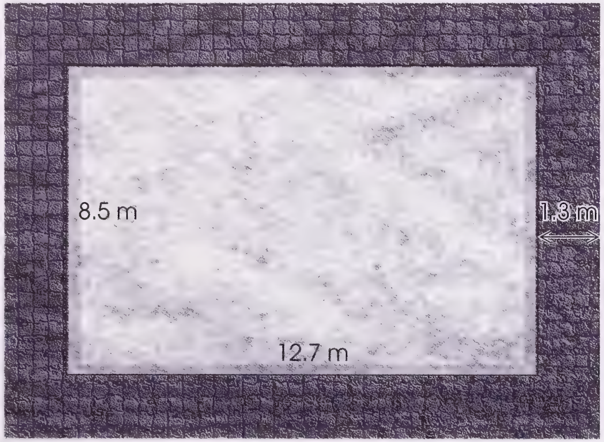
e. If $\sqrt{25} = 5$, then $\sqrt{25\,000\,000} =$ ____.

Check your answers by turning to the Appendix.



Activity 3: Using a Calculator Efficiently

A patio surrounding a rectangular swimming pool is of uniform width. Given the dimensions shown, find the area of the patio.



Do you know how to use a calculator to find the answer to this problem? Can you use a calculator in more than one way to solve this question?

In this activity you will use a number of different calculator methods to solve problems and discover which methods are the most efficient.

Look at the swimming pool question again. You can write out the calculation that needs to be done using a pencil and paper.

$$\begin{aligned}\text{Area of patio} &= (\text{Area of pool and patio}) - (\text{Area of pool})\\ &= (12.7 + 2.6) \times (8.5 + 2.6) - (12.7 \times 8.5)\end{aligned}$$

There is more than one way to do this calculation using a calculator.

Method 1

1 2 • 7 + 2 × 1 • 3 =

× (8 • 5 + 2 × 1 • 3)

= − 1 2 • 7 × 8 • 5 =

1. Enter the keystrokes in Method 1, and indicate what each operator is doing. **Note:** Since some keys vary from one calculator to the next, you may have to refer to the owner's manual.

Check your answers by turning to the Appendix.

Method 2

1 2 • 7 × 8 • 5
= Min 1 2 • 7 + 2
× 1 • 3 = × (8 • 5
+ 2 × 1 • 3) = − MR =

Note: Some calculators may have STO or M+ in place of Min and MRC in place of MR.

2. Enter the keystrokes in Method 2, and write out what each operator is doing.

Check your answers by turning to the Appendix.

Method 3

8 • 5 + 2 × 1 • 3 = Min
1 2 • 7 + 2 × 1 • 3 =
× MR − 1 2 • 7 × 8 • 5 =

3. Enter the keystrokes in Method 3, and write out what each operator is doing.

4. How many keystrokes are used by each method? **Note:** Be sure to count each digit and decimal entry as a keystroke.

5. Which method is the least efficient? Why?

6. What is the difference between Method 1 and Method 3?

7. Which method do you prefer?

8. Margo won a contest and has to answer the following skill-testing question.

$$6.1 - 3.2 \times 5.3$$

Margo tried each of the following keystrokes on her calculator to find an exact answer. Keystrokes A, B, and C gave the correct answer while keystrokes D and E gave the wrong answer.

A. 3 • 2 × 5 • 3
= Min 6 • 1 − MR =

B. 6 • 1 − 3 • 2
× 5 • 3 =

C. [3] [•] [2] [×] [5] [•] [3] [+/−]
[+] [6] [•] [1] [=]

D. [6] [•] [1] [−] [3] [•] [2] [=]
[×] [5] [•] [3] [=]

E. [3] [•] [2] [×] [5] [•] [3]
[÷] [6] [•] [1] [=]

a. Which keystroke sequence do you think is the most efficient? Why?

b. The keystroke sequences in D and E gave the wrong answer. Why?

9. Describe how you would use the memory input and memory recall keys on a calculator to evaluate the following:

$$(28.2+18.1)\div(2.6+19.3)$$

10. a. Write out the keystrokes you would use to evaluate the expression in question 9 using brackets.

b. What keystroke can be used to replace the first set of brackets?

11. Use a calculator to evaluate the following expression in two ways. Round your answer to one decimal place. Estimate your answer first.

$$\frac{24.6+31.7}{8.8+4.3-7.1}$$

12. Did you use at least one of the two methods given in the answer to question 11? If not, was one or more of your methods more or less efficient than the methods given in the answer to question 11?

Check your answers by turning to the Appendix.

Did you use the reciprocal function [$\frac{1}{x}$] in one of your methods for question 11? The reciprocal method is another way to evaluate expression involving division. To evaluate the expression in question 11 using the reciprocal function, enter the following keystrokes.

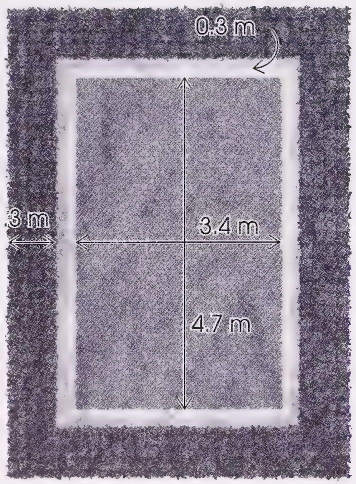
[8] [•] [8] [+] [4] [•] [3] [−] [7] [•] [1]
[=] [$\frac{1}{x}$] [×] [(] [2] [4] [•] [6]
[+] [3] [1] [•] [7] [)] [=]

13. What did the reciprocal function do? Is the answer the same?

14. What operation change was made to compensate for this change?

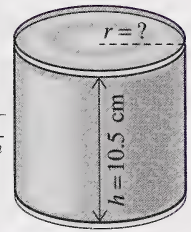
15. Is this method more efficient than the one given in the answer to question 11? Explain.

16. Stephan and Andrea are installing new carpet in their living room with dimensions 6.6 m by 7.9 m. They want a pattern with three different shades like that in the diagram. Calculate the area of each shade of carpet using an efficient calculator method. Estimate the areas first.



17. A can has a height of 10.5 cm and a volume of $490.7\ cm^3$. Use the given formula and the diagram to calculate the radius of the can. Use the following keys in your method. Round your answer to one decimal place.

$$r = \sqrt{\frac{V}{\pi h}}$$



$V = 490.7\ cm^3$

a. brackets and equals key

b. reciprocal key and equals key

18. Which method in question 17 is more efficient in terms of number of keystrokes?

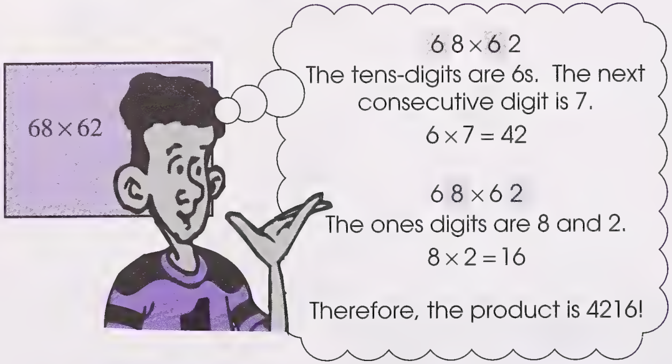
Check your answers by turning to the Appendix.

Did You Know?

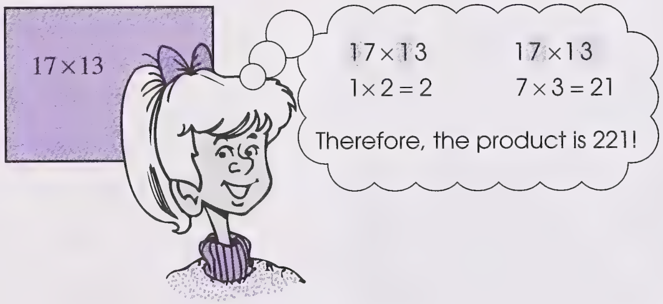
There are shortcuts to many calculations that allow you to do them faster mentally than if you were using a calculator. For instance, you can tell immediately that 56×54 is equal to 3024 if you know the secret.

You can multiply any pair of two-digit numbers whose tens digits are the same and whose ones digits total ten. By multiplying the tens digit by the next consecutive number, you get the first part of the product. The last two digits of the product are the product of the two ones digits.

Example 1



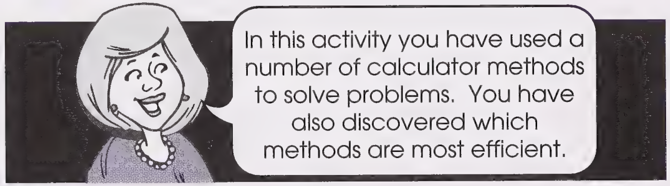
Example 2



19. Use the shortcut to mentally find the products of the following pairs of numbers.

a. 83×87	**b.** 94×96	**c.** 75×75
d. 31×39	**e.** 42×48	**f.** 53×57

Check your answers by turning to the Appendix.



Follow-up Activities

If you had difficulties understanding the concepts in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

You have seen from the exercises in this section that there are several ways of using a scientific calculator when performing multiple-step arithmetic calculations.

1. Copy and complete the following chart. The chart shows different methods of evaluating a given expression using a calculator.

Expression $\dfrac{204.41}{3.4 \times (18.7 - 12.5)}$	
Keystrokes	**Description**
1 8 • 7 − 1 2 • 5 = × 3 • 4 = Min 2 0 4 • 4 1 ÷ MR =	
	Evaluate the denominator first using equal keys; then multiply its reciprocal by the numerator.
2 0 4 • 4 1 ÷ (3 • 4 × (1 8 • 7 − 1 2 • 5)) =	
	Evaluate the denominator first using brackets; then use the memory keys to solve the expression.
(3 • 4 × (1 8 • 7 − 1 2 • 5)) $\frac{1}{x}$ × 2 0 4 • 4 1 =	

2. Use a calculator and any method from question 1 to evaluate the following:

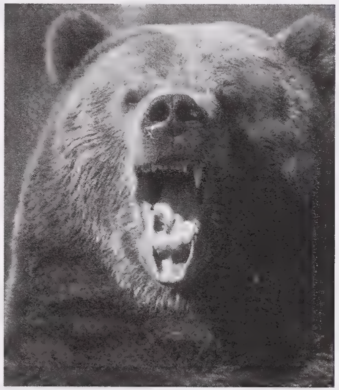
a. $(3.1-4.5)\times(6.2+1.4)$

b. $\dfrac{9.34\times 5.78}{-8.74+5.31}$

c. $\dfrac{3.1\times(8.7-5.9+3.1)}{9.8\times(17.1-10.7)}$

Check your answers by turning to the Appendix.

Enrichment



A ferocious brown bear isn't something most people want to meet face to face. Similarly, a **bear market** isn't something most investors look favourably upon. A bear market is characterized by falling prices and a negative outlook.

Rational numbers in all of the various forms (such as decimals, percents, fractions, and mixed numbers) are used in the stock market. The Canadian Stock Exchange is now using decimal numbers in its daily stock quotations. The following project gives you an opportunity to use various rational numbers while participating in an interesting activity.

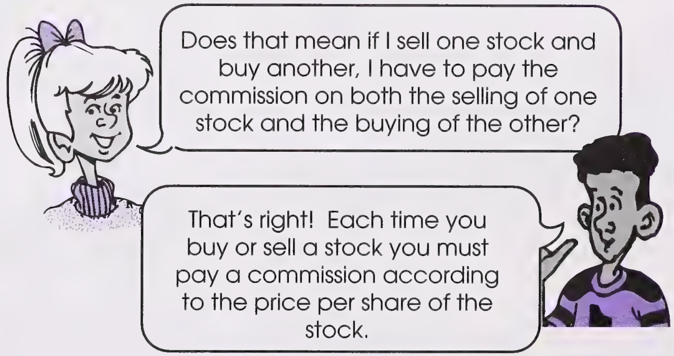
Playing the Stock Market

You have $10 000 to invest in the stock market. You must own shares in at least three companies at all times to a maximum of five companies at any one time.

Use any stock quotations given in a newspaper or on the Internet. You will have two weeks to earn as much money as possible through your investments.

You may wish to ask a friend or family member to do the same and see who can earn more over the two weeks.

Shares can be bought or sold anytime; but remember, you have to pay a commission each time you buy or sell stock.



Use the following chart to calculate the amount of commission you have to pay for each stock purchased or sold. **Note:** This commission chart is for Canadian Stock Markets. So, you must choose your stocks from one of the Canadian Stock Exchanges. The minimum for trading even one share of any price stock is $25.

Base Commission	Additional Cost/Share	Share Price
$25	0.5¢	0–$1.00
$25	2¢	$1.01–$5.00
$25	3¢	$5.01–$10.00
$25	4¢	$10.01–$20.00
$25	5¢	$20.01–$30.00
$25	6¢	over $30.00

Keep track of each stock on a separate chart similar to the following.

Company X

Date	Number of Shares	Price	Change from Previous Day	Amount Paid	Amount Received	Commission Paid
Mar. 12	200 (bought)	6.75		$1350.00		$31.00
Mar. 13	200 (bought)	7.25	0.50	$1450.00		$31.00
Mar. 14		7.50	0.25			
Mar. 15	200 (sold)	7.90	—		$1580.00	$31.00

You must be sure to keep track of how much money you have in total, especially if you have some losses, so you do not run out of money.

If you have access to a computer and a spreadsheet program, then set up the charts in the spreadsheet program.

At the end of the two-week period you should have the following:

- a completed chart for each of the stocks you purchased
- the amount of money you have left at the end of the project

You may make a graph of date versus price for each stock showing the daily changes.

In addition, answer the following questions.

1. Why did you choose the stocks that you did?

2. What are possible reasons for major fluctuations in the value of your stock?

3. Is the stock market a good place to invest your money? Explain.

Check your answers by turning to the Appendix.

Conclusion

This section has given you the opportunity to practise your calculator skills and problem-solving skills with rational numbers.

You used the calculator to help you solve various single-operation and multiple-operation problems and to determine which calculator methods were most efficient. You also used estimation to help you decide if your calculator answer was reasonable.

Were you surprised to find that your calculator answer may not always be correct? Do you feel more confident in using your calculator for multiple-step expressions and problems? Understanding calculators and computers will provide you with a greater number of careers (like that of a stock market analyst) in which to pursue.

Assignment

Assignment Booklet

You are now ready to complete the assignment for Section 1.

Section 2: Numbers and the Number System

NASA

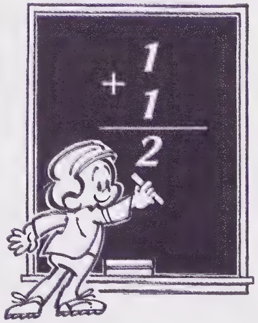
Are you familiar with different systems? For instance, your body is made up of several systems—the circulatory, muscular, skeletal, reproductive, nervous, and digestive systems. There are communication and transportation systems all around you. Earth itself is a very small part of the solar system, which, in turn, is part of an even larger system. All systems are made up of many interlocking or connecting parts.

Mathematics is also made up of many systems. The number system that you use is just one example. The number system consists of ways of naming and writing numbers; it is concerned with the relationships and interactions of its members.

In this section you will analyse four groups of the number system—natural numbers, whole numbers, integers, and rational numbers—and study their make-up and their relationship to each other. You will also apply rational numbers with the arithmetic operations and in some problem-solving situations.

Activity 1: Classifying Numbers

Happy, joyous, delighted, and pleased are all words that express a similar emotion. In mathematics, the same is true; many different forms can be used to express the same idea or quantity. In this activity you will identify numbers and express them in different forms.



Since your first day in school you have been working with various types of numbers. You started with the counting numbers or **natural numbers**. These numbers allowed you to count your crayons, blocks, candies, and so on.

1. The dots on this number line show some of the counting numbers.

   a. Is there a least number in this group?

   b. Is there a greatest number in this group?

   c. Are there natural numbers between 1 and 2, 2 and 3, 3 and 4, and so on?

Check your answers by turning to the Appendix.

Later, when you had no crayons or candies, you were informed that this could be represented by the number zero.

Whole numbers include the counting numbers and zero.

2. The dots on this number line show some whole numbers.

   a. What number is a whole number but not a natural number?

   b. Is there a least whole number or a greatest whole number?

   c. Are there whole numbers between 0 and 1, 1 and 2, 2 and 3, and so on?

Check your answers by turning to the Appendix.

As your knowledge of numbers grew, you began using another type of number to describe certain situations. For example, using positive and negative numbers to refer to temperature.

Integers include all the whole numbers and their opposites.

3. The dots on the number line show some integers.

   a. Is there a least integer or a greatest integer?

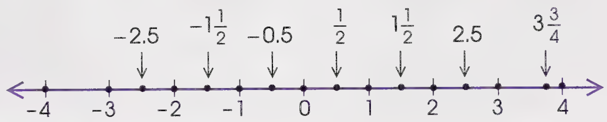
   b. Are there integers between -3 and -2, -2 and -1, -1 and 0, and so on?

Check your answers by turning to the Appendix.

As your experiences and knowledge of numbers continued to grow, you added positive and negative fractions and decimals. Thus, you were able to talk about a fraction of a pie or a race that was won by mere hundredths of a second.

Rational numbers include the counting numbers, zero, integers, positive and negative fractions, and positive and negative decimal numbers.

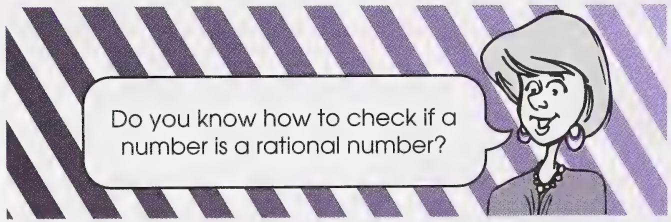
4. The dots on the number line show some rational numbers. Is there a least number or a greatest number?



Check your answers by turning to the Appendix.

The word "rational" comes from the Latin word *ratio*. A rational number can be expressed as a ratio of two integers in which the second integer is not equal to zero.

A rational number (Q) is any number that can be written as a fraction of the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.



The following points give you some criteria to follow when determining if a number is a rational number.

All natural numbers, whole numbers, and integers are rational numbers.

- 4 is a rational number since it can be written as $\frac{+4}{+1}$.
- 0 is a rational number since it can be written as $\frac{0}{+1}$.
- -2 is a rational number since it can be written as $\frac{-2}{+1}$.

All negative and positive fractions and mixed numbers are rational numbers.

- $\frac{1}{2}$ is a rational number since it can be written as $\frac{+1}{+2}$.
- $-\frac{3}{4}$ is a rational number since it can be written as $\frac{-3}{+4}$.
- $1\frac{1}{2}$ is a rational number since it can be written as $\frac{+3}{+2}$.
- $-1\frac{1}{2}$ is a rational number since it can be written as $\frac{-3}{+2}$.

All negative and positive terminating decimal numbers are rational numbers.

- 0.7 is a rational number since it can be written as $\frac{+7}{+10}$.
- -0.3 is a rational number since it can be written as $\frac{-3}{+10}$.
- 3.1 is a rational number since it can be written as $\frac{+31}{+10}$.
- -5.3 is a rational number since it can be written as $\frac{-53}{+10}$.

All negative and positive decimal numbers that have a repeating digit or a repeating block of digits are rational numbers.

- $0.\overline{63}$ is a rational number since it can be written as $\frac{+7}{+11}$.
- $-0.1\overline{6}$ is a rational number since it can be written as $\frac{-1}{+6}$.
- $4.\overline{3}$ is a rational number since it can be written as $\frac{+13}{+3}$.
- $-1.\overline{2}$ is a rational number since it can be written as $\frac{-11}{+9}$.

5. State if each of the following is **true** or **false**.

a. 4 is a whole number.

b. -4 is a natural number.

c. 0 is a rational number.

d. $-\frac{1}{2}$ is an integer.

e. 1.51 is an integer.

f. $-2\frac{3}{4}$ is a rational number.

g. -110 is an integer.

h. 5 is a whole number.

Check your answers by turning to the Appendix.

Not all decimal numbers repeat or terminate. Do you recall from Section 1 of this module what happened when you used a scientific calculator to find $\sqrt{2}$, $\sqrt{3}$, $\sqrt{22}$, and so on?

6. Use a scientific calculator to perform the indicated operation.

a. $2 \div 9$ **b.** $\sqrt{11}$ **c.** $\sqrt{89}$

d. $\sqrt{81}$ **e.** $\sqrt{50}$ **f.** $49 \div 6$

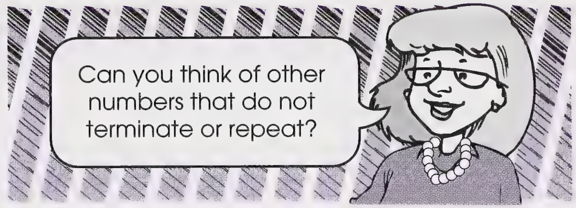
7. Do all the answers in question 6 repeat or terminate?

8. What has the calculator done with the answer to question 6.f.? How would you write the answer as a repeating decimal?

Check your answers by turning to the Appendix.

The decimal numbers that neither repeat nor terminate cannot be expressed as a ratio of two integers. These numbers are not rational numbers. They belong to a new group of numbers that are completely separate from the rational numbers.

- $-1.010\,010\,001\ldots$ is not a rational number since it cannot be written as $\frac{a}{b}$.
- $3.141\,592\,653\ldots$ is not a rational number since it cannot be written as $\frac{a}{b}$.



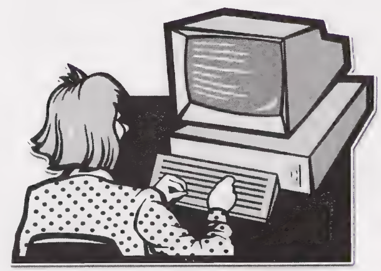
Use a scientific calculator to find the value for π. There should be a key that will give you the value for π on the calculator. The value of π is the ratio of the circumference of a circle to its diameter.

9. Does the value of π appear to be a terminating decimal or a repeating decimal? Is the value of π a rational number?

Check your answers by turning to the Appendix.

Did You Know?

Mathematicians, using modern computers, have calculated the value of π to millions of digits with no repeating sequence showing. Latest reports are that the value of π has been calculated to ten million digits.



Use the Internet, an electronic encyclopedia, or a regular encyclopedia to find the value of π. Check the digits to see if you can find a recurring repeating sequence. Enter the words "values of pi" on any of the Internet's search engines, or enter the following uniform resource locator (URL):

http://www.ccsf.caltech.edu/~roy/pi.html

Click on Digits to discover the first 50 000 digits of π.

Decimal numbers that neither repeat nor terminate are called irrational numbers and are not part of the set of rational numbers. You will study more about irrational numbers in Mathematics 10.

10. Explain why $\frac{8}{0}$ is not a rational number.

11. Explain why -8 is a rational number but not a whole number.

12. List at least three different situations that can be represented by rational numbers.

Check your answers by turning to the Appendix.

Applying Rational Numbers

Rational numbers can be written in various forms (such as fractions, decimals, and mixed numbers) and used in arithmetic operations performed on a calculator or using paper and pencil. In addition, rational numbers can be applied to some problem-solving situations.

Example 1

Calculate $\frac{7}{8} - 0.4$. Change the form of the number when necessary.

Solution

Enter $\frac{7}{8}$ as a division.

Example 2

Calculate $1\frac{3}{7} + 0.55$. Change the form of the number when necessary.

Solution

Separate the mixed number as 1 added to $\frac{3}{7}$. Complete the question in the order given.

1 + 3 ÷ 7 + • 5 5 =

1.978571429

Example 3

A farmer's sprayer tank will be filled using two separate lines. One line can fill the tank in 15 min, and the other line can fill the tank in 10 min. How long will it take to fill the sprayer tank if both lines are used?

Solution

The first line can fill the tank in 15 min; thus, it can fill $\frac{1}{15}$ of the tank in 1 min. The second line can fill the tank in 10 min; thus, it can fill $\frac{1}{10}$ of the tank in 1 min.

Therefore, both lines will fill $\frac{1}{15} + \frac{1}{10}$ of the tank in 1 min.

1 ÷ 1 5 + 1 ÷ 1 0 =

0.166666666

The two lines will fill $0.1\overline{6}$ of the tank in 1 min.

The farmer wants to fill the whole tank. Use 1 to represent the whole tank.

$$1 \div 0.1\overline{6} = 6$$

The two lines will fill the tank in 6 min.

To use the calculator in one sequence, enter the following keystrokes.

1 ÷ 1 5 + 1 ÷ 1 0 = $\frac{1}{x}$

6.

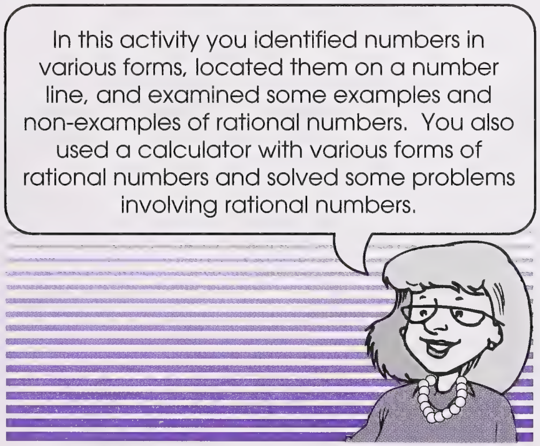
Use a calculator to answer questions 13 and 14.

13. Calculate the following to two decimal places.

a. $\frac{2}{3}+0.7$ **b.** $3\frac{1}{8}-1.74$ **c.** $4.15-2\frac{1}{6}$

14. A large vat will be filled using two taps. One tap can fill the vat in 12 min, and the other tap can fill the vat in 18 min. How long will it take both taps to fill the vat?

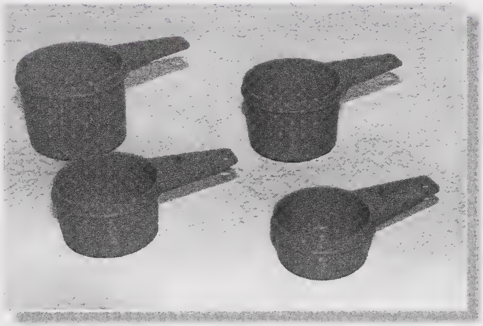
Check your answers by turning to the Appendix.



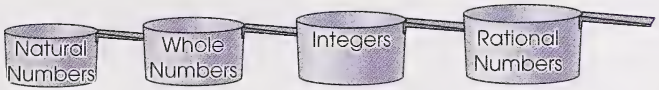
Activity 2: Understanding the Number System

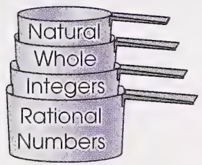
Are you familiar with nested dolls or a set of measuring cups? The individual dolls or cups fit together to make a set. The kinds of numbers you identified in Activity 1 of this section belong to different sets of numbers. Just like the dolls or the measuring cups, these sets of numbers fit one within the next and are part of the number system.

In this activity you will analyse how the parts of the number system fit together. You will see how the groups of numbers that you have studied in the previous activity fit together to become part of the number system.



You are most likely familiar with measuring cups used for baking. Find a set of four measuring cups, and examine how they fit together. Take the set apart and, using a piece of masking tape and a felt pen, label each one as shown in the diagram.





Now, put the measuring cups back together as a set. Study which labelled cup fits into which. This shows you which set of numbers is a part of another group. The natural numbers are a part of the whole numbers; the natural numbers and whole numbers are a part of the integers; and the natural numbers, whole numbers, and integers are part of the rational numbers.

1. Answer **yes** or **no** to each of the following questions. Explain your answer.

   a. Is the group of integers a part of the rational numbers?

   b. Is every rational number an integer?

   c. Is every whole number an integer?

   d. Is the group of integers infinite?

   e. Is every rational number either positive or negative?

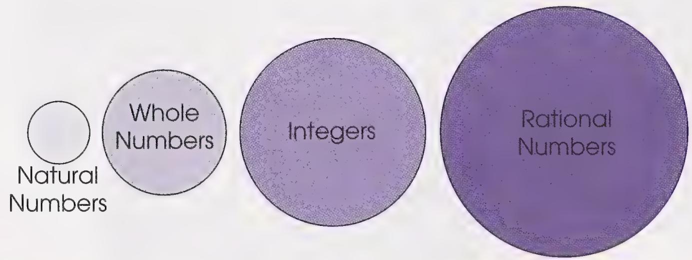
   f. Are some rational numbers integers?

   g. Give an example of a rational number that is an integer.

Check your answers by turning to the Appendix.

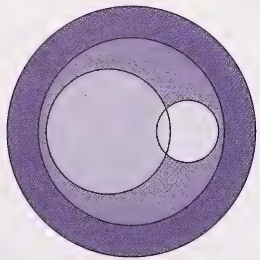
Another way to analyse the relationship between groups of numbers is through the use of a Venn diagram. A Venn diagram is a set of circles (or other shapes) used to represent number groups with different sizes of circles representing each group.

For example, a small circle could represent the natural numbers, a slightly larger circle could represent the whole numbers, and so on. The following circles represent the groups of numbers that you are familiar with.

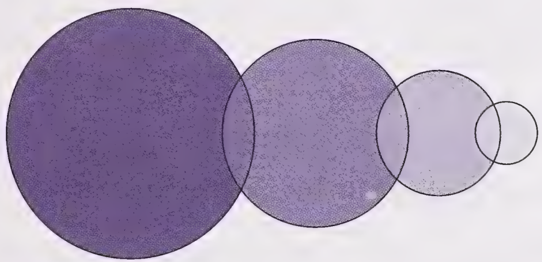


2. Which of the following diagrams shows the correct relationship among natural numbers, whole numbers, integers, and rational numbers? Each circle represents one of the four sets of numbers. Sketch your choice, and place the symbol for each set (*N*, *W*, *I*, or *Q*) in the correct circle. Explain why you chose the diagram you did.

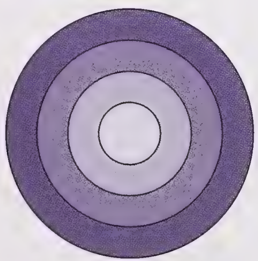
A.



B.



C.



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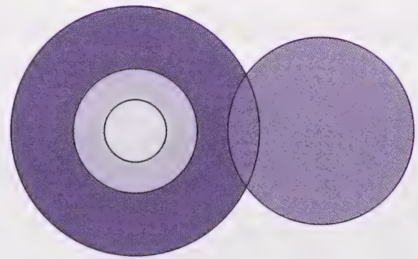


3. Classify the numbers 0, $-\frac{7}{8}$, 5, -4, and 1.25 as natural numbers, whole numbers, integers, or rational numbers.
Remember: The numbers may be classified in more than one group.

4. Explain why 5 is a natural number, a whole number, an integer, and a rational number.

5. Give an example of each of the following.

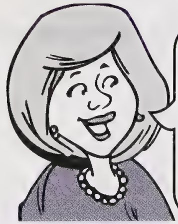
   a. an integer that is also a whole number

   b. a whole number that is not a natural number

   c. a rational number that is not an integer

   d. a rational number that is also a whole number

Check your answers by turning to the Appendix.



In this activity you have analysed how the various groups of numbers you have studied so far fit together within the number system. Were you surprised to discover that some numbers could belong to all the groups?

Follow-up Activities

If you had difficulties understanding the concepts in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

In this section you discovered that groups of numbers are nested or fit one within the next. Meaning, numbers can belong to one or more groups. You found that all the groups of numbers you studied thus far are rational numbers; but there are also some numbers that do not fit the definition of a rational number.

1. Copy the following chart into your notebook. Then complete the chart by putting a check mark in the appropriate column for the group of numbers that each number belongs to. The first one has been done for you.

Number	Type of Number			
	N	*W*	*I*	*Q*
-3			✔	✔
0				
$\frac{3}{5}$				
16				
$\sqrt{25}$				
$-\frac{11}{16}$				
$0.\overline{3}$				
$2\frac{2}{3}$				
-1.5				
$1.404\,004\ldots$				
-9				
$\sqrt{5}$				

2. Match each set of numbers with its description.

Description

i. set of counting numbers and zero

ii. set of numbers $\frac{a}{b}$, where a and b are integers and $b \neq 0$

iii. set of counting numbers

iv. set of whole numbers and their opposites

a. integers

b. whole numbers

c. natural numbers

d. rational numbers

Enrichment

Density Property of Rational Numbers

In Activity 1 of this section you studied number lines for each set of numbers studied. Go back and review those number lines; then compare the rational numbers with the natural numbers, whole numbers, and integers.

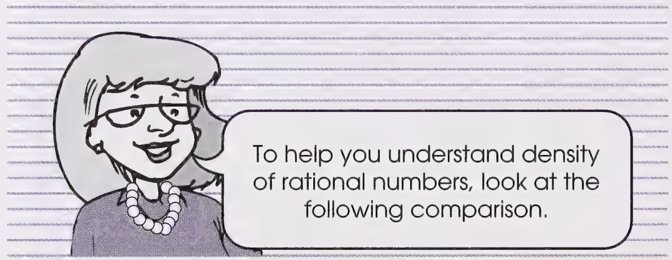
1. What do you notice about the number of dots there are on the rational number line compared to the other number lines?

2. Can you find a rational number between $-1\frac{1}{2}$ and -1 or between $\frac{1}{2}$ and 1? Give an example of each.

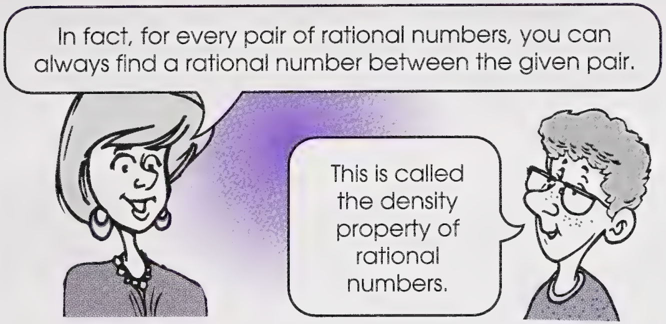
3. Can you find more than one rational number between the pairs of numbers given in question 2?

4. What does this indicate about rational numbers?

Check your answers by turning to the Appendix.



Are you familiar with high-density computer disks? A high-density computer disk is one that has a large number of places to store information. This is similar to the rational number line where there is a large number of places to have rational numbers.



The density property of rational numbers states that there is at least one rational number between any two rational numbers.

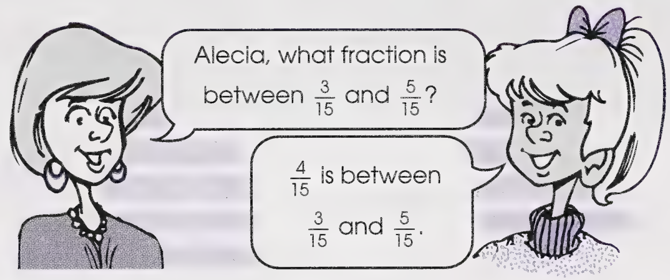
Example 1

Find a rational number between $\frac{1}{5}$ and $\frac{1}{3}$.

Solution

Change $\frac{1}{5}$ and $\frac{1}{3}$ to common denominators.

$$\frac{1}{5}=\frac{3}{15} \qquad \frac{1}{3}=\frac{5}{15}$$



Thus, $\frac{4}{15}$ is a rational number between $\frac{1}{5}$ and $\frac{1}{3}$.

Example 2

Find a rational number between $-\frac{3}{4}$ and $-\frac{2}{3}$.

Solution

Change $-\frac{3}{4}$ and $-\frac{2}{3}$ to common denominators.

$$-\frac{3}{4}=-\frac{9}{12} \qquad -\frac{2}{3}=-\frac{8}{12}$$

There is no integer between -9 and -8; so, you need to find equivalent fractions with a larger denominator.

$$-\frac{9}{12}=-\frac{18}{24} \qquad -\frac{8}{12}=-\frac{16}{24}$$

Thus, $-\frac{17}{24}$ is between $-\frac{18}{24}$ (or $-\frac{3}{4}$) and $-\frac{16}{24}$ (or $-\frac{2}{3}$).

Example 3

Find a rational number between -0.19 and -0.18.

Solution

Write equivalent decimal numbers by adding a zero to each.

$$-0.19 = -0.190 \qquad -0.18 = -0.180$$

Some numbers between -0.19 and -0.18 are
$-0.189,\ -0.188,\ -0.187, \ldots,\ -0.183,\ -0.182$, and -0.181.

5. Write two rational numbers that come between each of the following pairs of numbers.

a. $\frac{3}{8}, \frac{5}{12}$ **b.** 1.6, 1.65 **c.** $-0.4,\ -0.3$

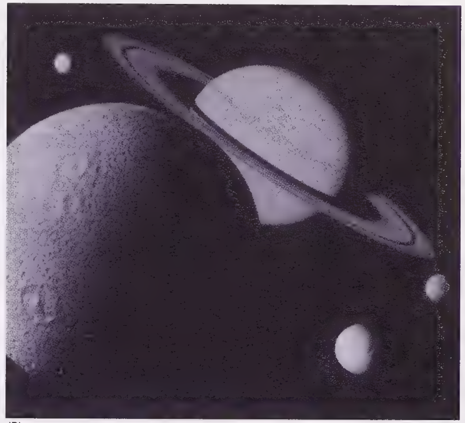
d. $4.3,\ 4.\overline{3}$ **e.** $-\frac{5}{6}, -\frac{7}{9}$ **f.** $\frac{5}{12}, \frac{2}{3}$

6. How would you find a rational number exactly halfway between two other rational numbers?

Check your answers by turning to the Appendix.

Conclusion

This section helped you identify what numbers make up the various number groups. It gave you an understanding of how the groups relate to each other and fit within the entire number system.



JPL

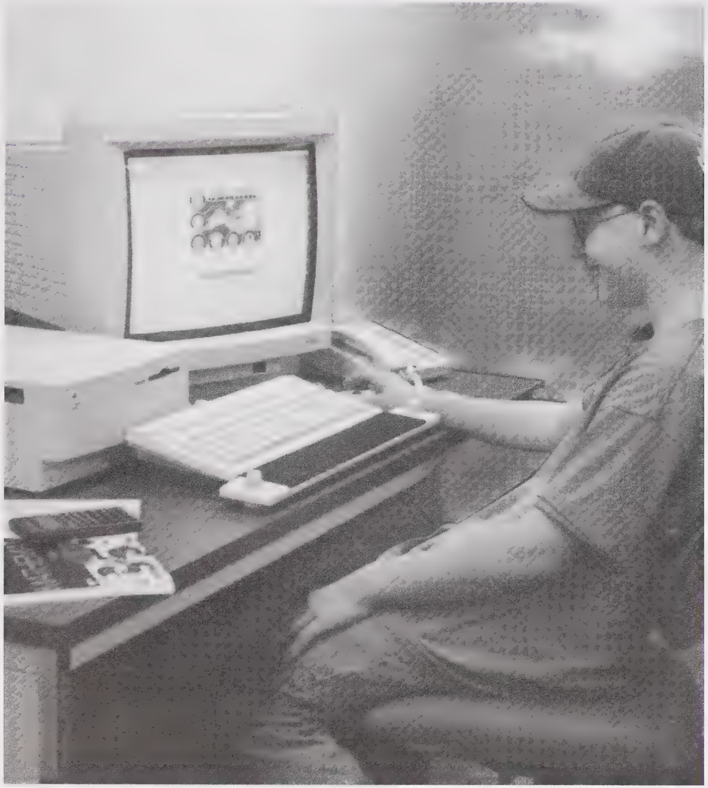
You identified four groups of numbers—natural numbers, whole numbers, integers, and rational numbers—and you identified specific numbers within each of these groups. You also identified some numbers that did not fit in any of the four groups of numbers you studied.

Were you surprised to discover that there are more numbers than those that belong to the four groups you studied? The number system, like the solar system, has many other parts that have not been explored yet.

Assignment Booklet

You are now ready to complete the assignment for Section 2.

Module Summary



This module helped you to develop a sense of confidence in the use of calculators in problem solving. In Section 1 you used a calculator to add, subtract, multiply, and divide rational numbers. You were also shown how estimating is valuable and helps keep a check on your calculator skills.

This module also gave you an understanding of the number system. In Section 2 you classified numbers into four different groups. Then, you analysed how these various groups of numbers relate to each other and fit within the number system. You were to use your calculator to help you identify which numbers fit into the individual groups.

Do you feel more confident in using a calculator in mathematics now? Did you have the opportunity to use a computer as well? If not, do you believe you are ready to use a computer in future studies of mathematics?

Final Module Assignment

Assignment Booklet

You are now ready to complete the final module assignment.

APPENDIX

Problem-Solving Strategies

Glossary

Suggested Answers

Problem-Solving Strategies

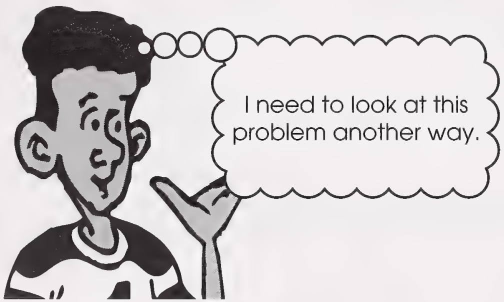
Be sure to study the following problem-solving strategies and use them when you encounter problems in this course.

Whichever method of problem solving you may choose to use, consider the following:

- Show that you understand the problem by showing all the steps needed for finding the answer.
- Draw and neatly label any diagrams, graphs, or charts that may help you answer the problem.
- Write a concluding sentence that answers the question being asked.

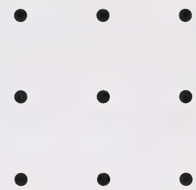
Changing Your Point of View

Sometimes you find that you cannot solve a particular problem because of your "mind set." Perhaps you made an assumption about the problem that is incorrect. If your attempts to solve a problem are not being successful, it is often helpful to try to change your point of view.



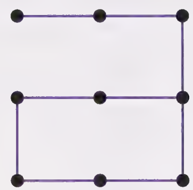
Example

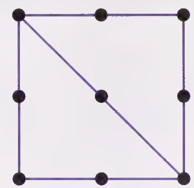
Draw four line segments through the following nine dots without lifting your pencil from the paper and without retracing your path.

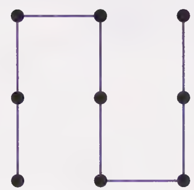


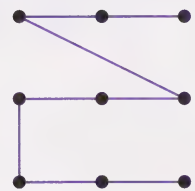
Solution

Step 1: You make several attempts at solving the problem.


5 line segments

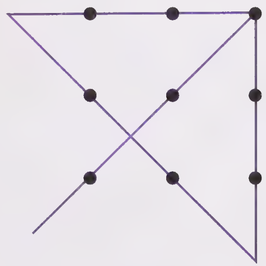

5 line segments


5 line segments


5 line segments

Step 2: You realize you are blocked in your attempts. You have made the assumption that the dots must be connected in a way similar to a child's dot-to-dot picture.

In order to solve the problem you must change your point of view and realize that the line segments may lie outside the confines of the dots.



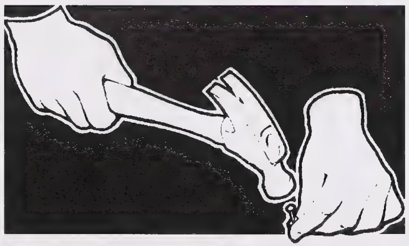
4 line segments

Using Objects

Objects can be used to help you solve a problem.

Example 1

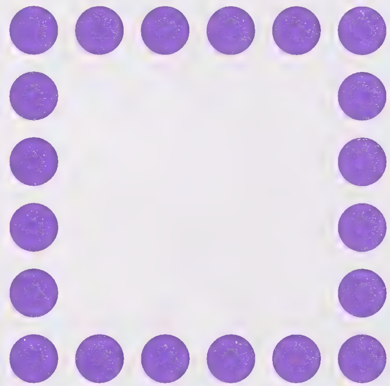
To enclose a square lot with a fence, 20 posts are used. If the posts are placed 3 m apart, what is the length of one side of the lot?



Solution

Step 1: Use objects such as blocks, checkers, or coins to represent the posts.

The advantage of using objects to represent the posts is that you can easily rearrange them.



Step 2: Count the number of spaces between the posts on one side of the square.

There are five spaces between the six posts on one side of the square.

Step 3: Calculate the length of one side of the square.

$$5 \times 3 = 15$$

The length of one side of the lot is 15 m.

Using Diagrams

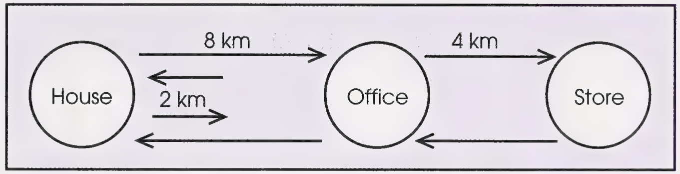
You can use sketches to solve problems.

Example 1

Chris' office is 8 km from his home. Yesterday morning he drove 2 km before he realized that he had forgotten his briefcase. He returned home to get his briefcase and then drove to his office. At noon Chris drove 4 km to a store. He then went back to the office for the rest of the afternoon. At the end of the day he drove straight home. How far did he drive yesterday?

Solution

Step 1: Draw and label a diagram to help you understand the problem.



Step 2: Calculate the distance Chris drove.

$$2 + 2 + 8 + 4 + 4 + 8 = 28$$

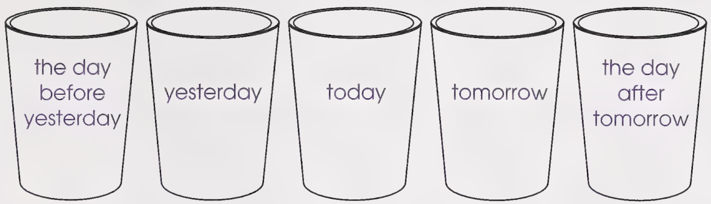
Chris drove 28 km.

Example 2

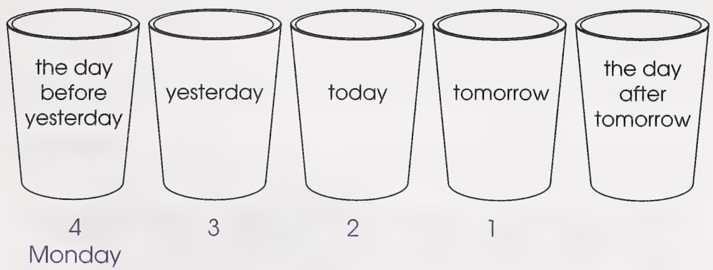
Four days before the day after tomorrow was Monday. What day of the week is it today?

Solution

Step 1: Use paper cups to represent the days. The cups could be labelled as shown.



Step 2: Begin at the cup representing the day after tomorrow and count back four days. That day was Monday.



If Monday was the day before yesterday, yesterday was Tuesday, and today is Wednesday.

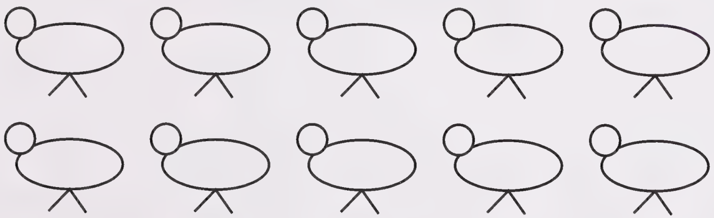
Example 2

There are ten animals in a barnyard. Some are chickens and the rest are sheep. Ruth counts 28 legs. How many chickens and how many sheep are there? **Hint:** Chickens have 2 legs and sheep have 4 legs.

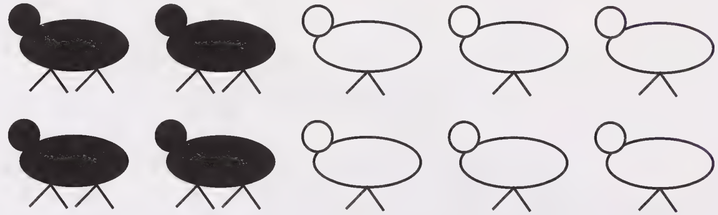
Solution

Draw a diagram to help you solve the problem.

Step 1: Every animal has at least 2 legs. So, draw a diagram of ten animals, each of which has at least 2 legs.



Step 2: The group of animals in the problem had 28 legs. So, you need to add 8 legs to the drawing. Add the legs in pairs to the sketch of each animal.



There are four sheep and six chickens.

Making an Organized List

Some problems require you to list all the possible solutions. It is important to make an organized list so that you do not miss any possibilities.

Example 1

Julio and his friends like playing a game called Bull's Eye with lawn darts. They draw a target in the dirt and assign values to the three regions of the target.

If Julio throws three darts and all three hit the target, how many different point totals are possible?

Solution

Begin with the highest possible score and list all the possibilities in order.

$9+9+9=27$ ← highest possible score
$9+9+4=22$
$9+9+3=21$
$9+4+4=17$
$9+4+3=16$
$9+3+3=15$
$4+4+4=12$
$4+4+3=11$
$4+3+3=10$
$3+3+3=9$ ← lowest possible score

There are ten possible point totals.

Example 2

At an ice-cream parlour there are three flavours of ice cream. How many different double-scoop ice-cream cones are possible? **Hint:** A chocolate-vanilla cone is different from a vanilla-chocolate cone. Order makes a difference.

Solution

Draw a tree diagram to help you make an organized list. Use V for vanilla, C for chocolate, and S for strawberry.

Scoop 1	Scoop 2	Result
V	V	(V, V)
	C	(V, C)
	S	(V, S)
C	V	(C, V)
	C	(C, C)
	S	(C, S)
S	V	(S, V)
	C	(S, C)
	S	(S, S)

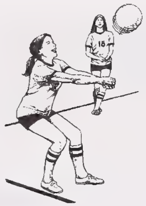
Nine different double-scoop ice-cream cones are possible.

Using Venn Diagrams

A type of diagram that is helpful in some problems is a Venn diagram. In this special kind of diagram, circles are used to represent groups of people, animals, or objects that have certain characteristics. The positioning of the circles in relation to one another represents relationships among these groups. These diagrams were named after an English mathematician named John Venn (1834–1923).

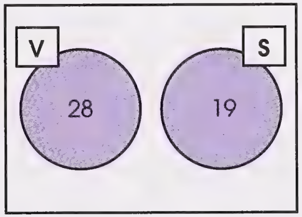
Example 1

There are 28 girls on the volleyball team and 19 girls on the swim team. If 12 girls belong to both teams, how many girls belong only to the swim team?

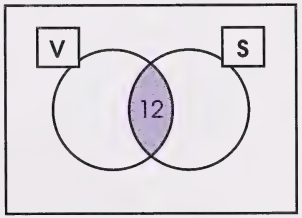


Solution

Step 1: Represent the members of the volleyball team with a circle. Represent the members of the swim team with another circle.

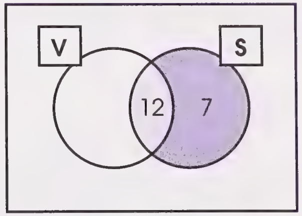


Step 2: Represent the members of both teams by the intersection of these two circles.



Step 3: Now calculate the number of girls who belong only to the swim team.

$$19 - 12 = 7$$



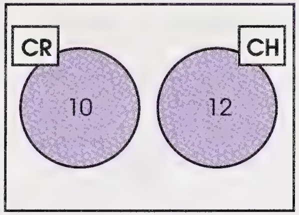
So, 7 girls belong only to the swim team.

Example 2

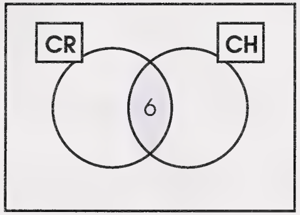
Reilly bought some plates at a garage sale. He discovered that 10 plates were cracked, 12 plates were chipped, 6 plates were both chipped and cracked, and 2 plates were neither chipped nor cracked. How many plates did Reilly buy?

Solution

Step 1: The problem states 10 plates were cracked and 12 plates were chipped. Represent the cracked plates with a circle. Represent the chipped plates with a second circle.

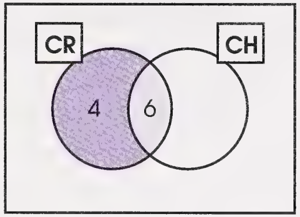


Step 2: The problem states that 6 plates were both chipped and cracked. Represent the cracked and chipped plates with the intersection of the two circles.



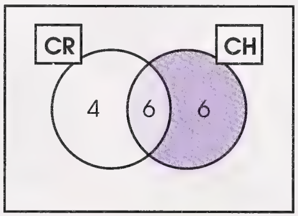
Step 3: Calculate the number of plates that were only cracked.

$$10 - 6 = 4$$

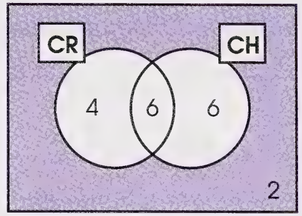


Step 4: Calculate the number of plates that were only chipped.

$$12 - 6 = 6$$



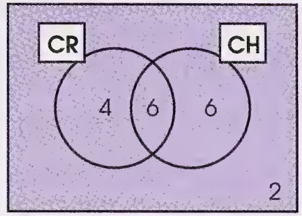
Step 5: The problem states that 2 plates were neither chipped nor cracked. Represent the undamaged plates with the region outside the circles.



Step 6: Calculate the total number of plates that were bought.

$$4 + 6 + 6 + 2 = 18$$

So, 18 plates were bought.



Example 3

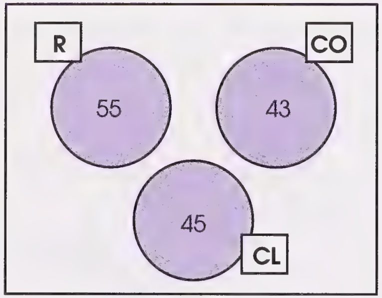
In a survey of 100 people, the following information was collected:

- 43 people like country music
- 55 like rock music
- 45 like classical music
- 8 people like all three forms
- 15 people like both country music and classical music
- 25 like both rock and country music
- 20 like both rock and classical music

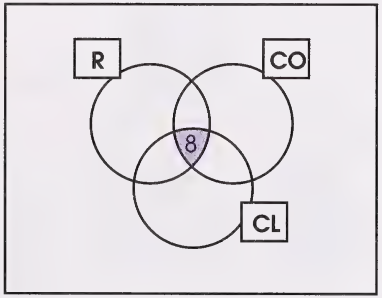
How many people do not like any of these forms of music?

Solution

Step 1: The problem states that 55 people like rock music, 43 like country music, and 45 like classical music. Use three circles to represent these preferences.

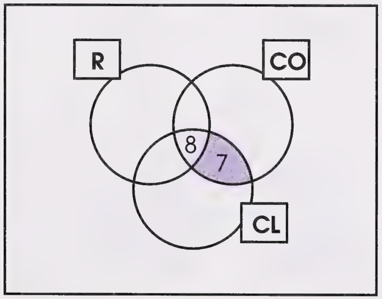


Step 2: The problem states that 8 people like all three forms. The intersection of the three circles shows the people who like all three forms.



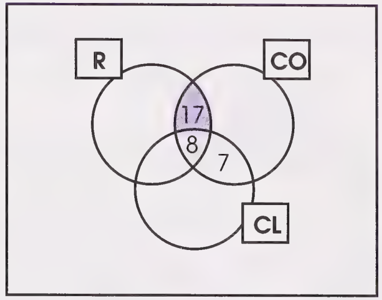
Step 3: The problem states that 15 people like both country music and classical music. Calculate the number of people who like country and classical music, but do not like rock music.

$$15 - 8 = 7$$



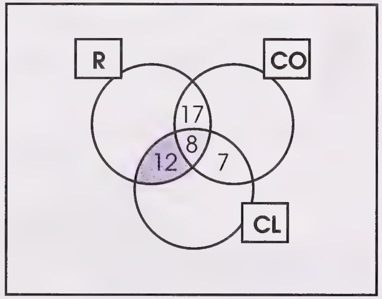
Step 4: The problem states that 25 people like both rock and country music. Calculate the number of people who like both country and rock music, but do not like classical music.

$$25 - 8 = 17$$



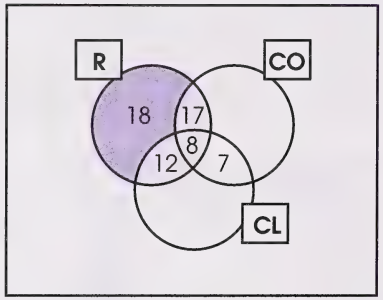
Step 5: The problem states that 20 people like both rock and classical music. Calculate the number of people who like both rock and classical music, but not country music.

$$20 - 8 = 12$$



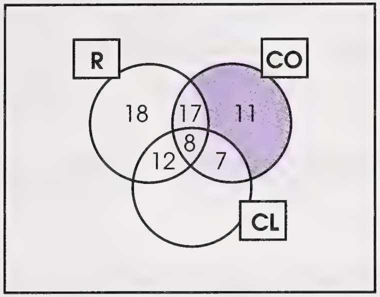
Step 6: Now that you know the number of people who like rock music and another form of music, calculate the number of people who only like rock.

$$55 - (17 + 8 + 12) = 18$$



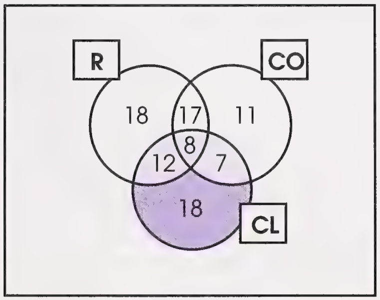
Step 7: Now that you know the number of people who like country music and another form of music, calculate the number of people who only like country music.

$$43-(17+8+7)=11$$



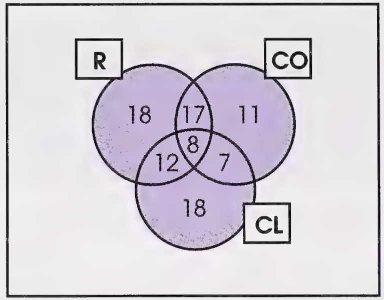
Step 8: Now that you know the number of people who like classical music and another form of music, calculate the number of people who like only classical music.

$$45-(12+8+7)=18$$



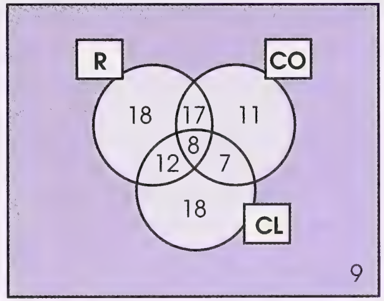
Step 9: Now you can calculate the total number of people who like rock, country, or classical music.

$$18+17+8+12+11+7+18=91$$



Step 10: The problem states that 100 people were surveyed. So, you can calculate the number of people who do not like any of these forms of music.

$$100-91=9$$



Of the 100 people surveyed, 9 people do not like any of these forms of music.

Making a Table

You can use a table to organize information.

Example

Hannah works at her mother's store after school. She earns 20¢ for each customer that she assists and 35¢ for each bag that she packs. One day she did 19 jobs and earned $4.85. How many customers did she assist that day and how many bags did she pack?

Solution

Write down all possible combinations of 19 jobs until you find a combination that totals $4.85. Use a table to organize the information.

Packing	1	2	3	4	5	6	7
Assisting	18	17	16	15	14	13	12
Earned	$3.95	$4.10	$4.25	$4.40	$4.55	$4.70	$4.85

correct guess

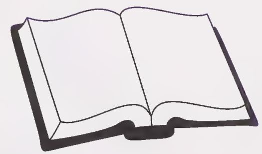
Hannah assisted 12 customers and packed 7 bags that day.

Guessing, Checking, and Revising

Many mathematical problems can be solved by systematic trial or guessing, checking, and revising.

Example

Shelley opened her math textbook and noticed that the product of the numbers of the two open pages was 2970. To what pages was the book opened? **Hint:** The left-hand page number of a book is even; the right-hand page number is odd.



Solution

Method 1: Using Paper and Pencil

Step 1: Note the conditions that need to be met.

- The left-hand page number must be even.
- The right-hand page number is one more than the left-hand page number.
- The product of the numbers must be 2970.

Step 2: Make a guess, check your guess, and revise if necessary.

It is helpful to organize your guesses in a table.

Left-hand page number	50	52	54
Right-hand page number	51	53	55
Product of page numbers	2550	2756	2970

correct guess

So, the book was open to pages 54 and 55.

Method 2: Using a Computer

Use a computer and a spreadsheet program to help you solve the problem.

Step 1: Label the cells on the spreadsheet. **Hint:** The cell address at the top of the spreadsheet shows which cell is active. There is also a border around the active cell. You don't type directly in this cell; instead you use the entry bar at the top of the spreadsheet.

A5 | × | ✓ | Product of page numbers

	A	B
1	Book Problem	
2		
3	Left-hand page number	
4	Right-hand page number	
5	Product of page numbers	
6		

Step 2: Enter either a number or a formula for each of the cells.

From the problem, you know that the right-hand page number is one more than the left-hand page number and that the product of the page numbers is 2970.

Here is what you enter:

- Begin with the B3 cell and enter a guess of 50.
- Next, enter = B 3 + 1 for the B4 cell. This means the number in the B4 cell will be one more than the number in the B3 cell.
- Finally, enter = B 3 * B 4 for the B5 cell. This means that the number in the B5 cell will be the product of the numbers in the B3 and B4 cells.

B5 | × | ✓ | =B3*B4

	A	B
1	Book Problem	
2		
3	Left-hand page number	50
4	Right-hand page number	51
5	Product of page numbers	2550
6		

The computer performs the calculations and the results appear in the B4 and B5 cells.

Step 3: Check your initial guess and revise it if necessary. The product is 2970, not 2550; therefore, revise your guess.

Delete the number in the B3 cell and enter 54.

B3 | × | √ | 54

	A	B
1	Book Problem	
2		
3	Left-hand page number	54
4	Right-hand page number	55
5	Product of page numbers	2970
6		

The computer performs the calculations and different results appear in the B4 and B5 cells.

Step 4: Check your initial guess and revise it if necessary. The product is 2970.

So, the book was open to pages 54 and 55.

Acting Out a Problem

For some problems, you may find it helpful to physically act out the problem situation.

Example

How many cards does each boy have?

Solution

Step 1: Ask yourself these questions:

- What would happen if Jon gave Matt a card? The first condition to be met is that they must have an equal number of cards after the exchange.
- What would happen if Matt gave Jon a card? The second condition to be met is that Jon will have twice as many cards as Matt after the exchange.

Step 2: Make a guess and act out the problem. If your guess does not fit the conditions, revise your guess and repeat the process.

It is helpful to organize your guesses in a table.

Matt's cards	1	2	3	4	5
Jon's cards	3	4	5	6	7

correct guess

So, Matt has 5 cards and Jon has 7 cards.

Note

If Jon gives Matt a card, each boy will have an equal number of cards after the exchange.

Each boy will have 6 cards.

If Matt gives Jon a card, Jon will have twice as many cards as Matt after the exchange.

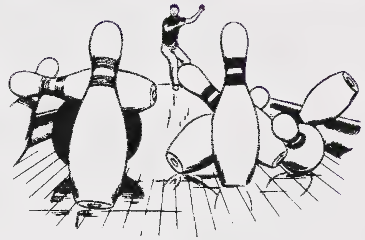
Jon will have 8 cards and Matt will have 4 cards.

Working Backwards

In mathematics you are usually given information, and then asked to find the answer. Sometimes, however, you are given the answer and then you are asked to find a piece of information. In cases like these, you need to work backwards.

Example 1

Ross plays in a bowling league. His average in the league is 198. Today Ross scored 195 and 175 in his first two games. What minimum score must he make in his third game to maintain his average?



Solution

Step 1: Consider how the problem would be done if you worked forwards. This flow chart shows the sequence of the steps.

Start ? → Add 195. → Add 175. → Divide by 3. → End 198

You start with the score of the third game, add each of the scores of the other two games, divide by 3, and end with the average of 198.

You do not know one of the three scores, so you cannot do the first step, but you can work backwards.

Step 2: Use a reverse flow chart to work backwards.

These operations undo operations in the original flow chart.

End ? ← Subtract 195. ← Subtract 175. ← Multiply by 3. ← Start 198

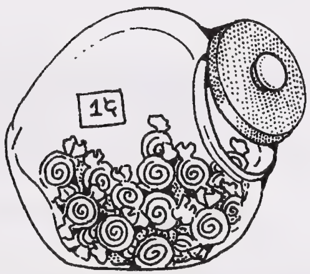
If you start with the average of 198, multiply by 3, subtract 175, and subtract 195, you will end with the minimum score that Ross must get in the third game.

Ross must get at least 224 in the last game.

Note: You can check to see if 224 is the correct third score by using the flow chart in Step 1.

Example 2

Ruth has a jar of candies. She gives Raschid one-third of her candies. She then gives Steven one-fourth of the remaining candies. Finally, she gives one-fifth of the candies that she has left to Rachel. If Ruth has 12 candies left at the end, how many did she have at the beginning?



Solution

Step 1: Use a flow chart to show the order of operations.

Start ? → Multiply by $\frac{1}{3}$. → Subtract ? → Multiply by $\frac{1}{4}$. → Subtract ? → Multiply by $\frac{1}{5}$. → Subtract ? → End 12

This flow chart is **not** helpful because you do not know how much to subtract in the second, fourth, and sixth boxes.

Knowing the following relationships will help you draw a more complete flow chart.

- Giving away one-third of the candies is the same as multiplying the number of candies by two-thirds.

$$1-\frac{1}{3}=\frac{2}{3}$$

- Giving away one-fourth of the remaining candies is the same as multiplying by three-fourths.

$$1-\frac{1}{4}=\frac{3}{4}$$

- Giving away one-fifth of the remaining candies is the same as multiplying by four-fifths.

$$1-\frac{1}{5}=\frac{4}{5}$$

The following flow chart is more useful.

Start ? → Multiply by $\frac{2}{3}$. → Multiply by $\frac{3}{4}$. → Multiply by $\frac{4}{5}$. → End 12

Step 2: Use a reverse flow chart to work backwards.

End ? ← Divide by $\frac{2}{3}$. ← Divide by $\frac{3}{4}$. ← Divide by $\frac{4}{5}$. ← Start 12

Ruth had 30 candies at the beginning.

Simplifying a Problem

Are you sometimes confused about which operations to perform in a problem because the numbers in the problem are very large or because the problem seems very complicated? There are strategies you can use to simplify a problem.

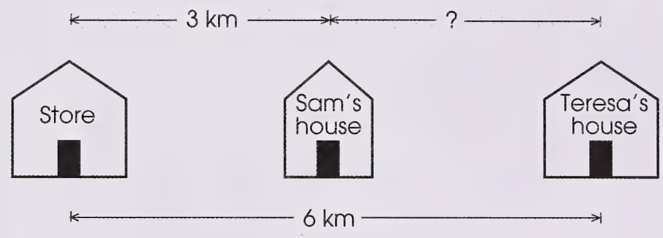
Example

The average distance from the Sun to Mars is 228 000 000 km and the average distance from the Sun to Earth is 150 000 000 km. What is the distance between Earth and Mars?

Solution

Step 1: If you find the large numbers overwhelming, make up a related but simpler problem.

Sam and Teresa both live east of the store. Sam lives 3 km from the store and Teresa lives 6 km from the store. How far is it from Sam's house to Teresa's house?



$6 - 3 = 3$

It is 3 km from Sam's house to Teresa's house.

Step 2: Once you understand the related but simpler problem, you can solve the original problem.

$$228\,000\,000 - 150\,000\,000 = 78\,000\,000$$

The distance from Earth to Mars is 78 000 000 km.

Finding and Applying a Pattern

To solve some problems, it is often helpful to use a related but simpler problem to discover a pattern.

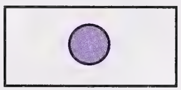
Example

There are 20 students in a room. If every student shakes hands with every other student in the room, how many handshakes are exchanged?

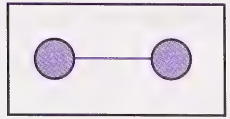
Solution

Step 1: Try solving the problem with 1, 2, 3, 4, or 5 students.

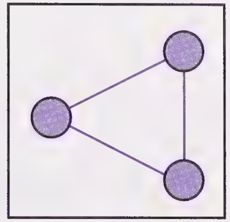
- If there is 1 student, no handshakes can be exchanged, as illustrated by the diagram at the right.



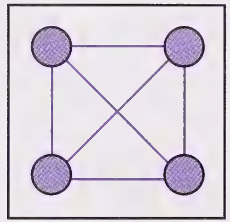
- If there are 2 students, 1 handshake can be exchanged, as illustrated by the diagram at the right.



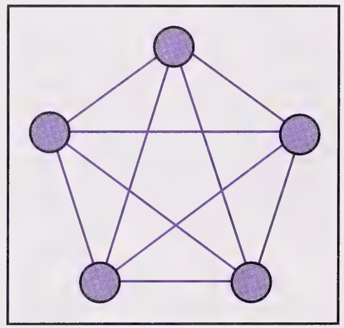
- If there are 3 students, 3 handshakes can be exchanged, as illustrated by the diagram at the right.



- If there are 4 students, 6 handshakes can be exchanged, as illustrated by the diagram at the right.



- If there are 5 students, 10 handshakes can be exchanged, as illustrated by the diagram at the right.



Step 2: Look for a pattern.

People	Handshakes	Pattern
1	0	
2	1	+1
3	3	+2
4	6	+3
5	10	+4

Step 3: Apply the pattern.

A calculator is helpful for applying the pattern.

1 + 2 + 3 + 4 + 5 + 6 +
7 + 8 + 9 + 10 + 11 +
12 + 13 + 14 + 15 +
16 + 17 + 18 + 19 =

190.

The total number of handshakes exchanged will be 190.

Using Elimination

Detectives are skilful at eliminating clues and solving mysteries. You can become skilful at using elimination to solve problems too.

Example

Five animals had a race over a short distance. Neither the donkey nor the coyote beat the lion. The coyote beat the rabbit, but not the ostrich. The donkey lagged behind the rabbit. If the ostrich was not the fastest, in what order did the animals finish the race?

Solution

Step 1: Decide what clues are given. The clues indicate the following:

- The donkey and coyote did not finish first. They did not beat the lion.
- The rabbit did not finish first. The coyote beat it.
- The coyote did not finish first. The ostrich was faster.
- The donkey did not finish first. It lagged behind the rabbit.
- The ostrich did not finish first. It was not the fastest.

The clues can be shown on a table.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	
Second					
Third					
Fourth					
Fifth					

Step 2: Use elimination and reconsider the clues.

By elimination, it is clear that the lion was first. Now reconsider the clues:

- The rabbit did not finish second. The coyote beat it.
- The coyote did not finish second. The ostrich was faster.
- The donkey did not finish second. It lagged behind the rabbit.

The additional clues can be shown in the table.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	✓
Second	✗	✗	✗		✗
Third					✗
Fourth					✗
Fifth					✗

Step 3: Use elimination and reconsider the clues.

By elimination, it is clear that the ostrich finished second. Now reconsider the clues:

- The rabbit did not finish third. The coyote beat it.
- The donkey did not finish third. It lagged behind the rabbit.

The additional clues can be shown in the table.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	✓
Second	✗	✗	✗	✓	✗
Third	✗		✗	✗	✗
Fourth				✗	✗
Fifth				✗	✗

Step 4: Use elimination and reconsider the clues.

By elimination, it is clear that the coyote finished third. Now reconsider the clues:

- The donkey lagged behind the rabbit.

So, the rabbit was fourth and the donkey was fifth.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	✓
Second	✗	✗	✗	✓	✗
Third	✗	✓	✗	✗	✗
Fourth	✗	✗	✓	✗	✗
Fifth	✓	✗	✗	✗	✗

So, the lion was first, the ostrich was second, the coyote was third, the rabbit was fourth, and the donkey was last.

Using Truth Tables

A strategy that you can use to solve some logic problems is making truth tables. Truth tables are similar to elimination tables.

Example

Detective Beagle is investigating a robbery in the forest. Charlie Chipmunk and Sammy Squirrel each deny being the robber. Harold Hare blames Charlie Chipmunk. Detective Beagle knows two of the animals are lying and one is telling the truth. Who is the robber?

Solution

Step 1: Assume that Charlie Chipmunk was the robber.

- Charlie Chipmunk said he didn't do it. If Charlie Chipmunk was the robber, this is a false statement. Make a truth table and put **F** under Charlie's name.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F		

- Sammy Squirrel denied doing it. If Charlie Chipmunk was the robber, this is a true statement. Put **T** under Sammy's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	

- Harold Hare blamed Charlie Chipmunk. If Charlie Chipmunk was the robber, this is a true statement. Put **T** under Harold's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T

Detective Beagle knows two animals are lying. So, Charlie is not the robber.

Step 2: Assume that Sammy Squirrel was the robber.

- Charlie Chipmunk denied doing it. If Sammy Squirrel was the robber, this is a true statement. Put **T** under Charlie's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T
T		

- Sammy Squirrel denied doing it. If Sammy Squirrel was the robber, this is a false statement. Put **F** under Sammy's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T
T	F	

- Harold Hare blamed Charlie Chipmunk. If Sammy Squirrel was the robber, this is a false statement. Put **F** under Harold's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T
T	F	F

Detective Beagle knows two of the animals are lying and one is telling the truth.

So, Sammy was the robber.

Using an Equation

A strategy you can use to solve some problems is to write and solve an equation.

Example 1

Twelve less than a number is negative three. Find the number.

Solution

Step 1: Let the unknown be equal to a variable. Let the number be n.

Step 2: Write an equation.

$$n - 12 = -3$$

Twelve less than a number is negative three.

Step 3: Solve the equation.

$$\begin{array}{rcr} n - \cancel{12} & = & -3 \\ +\cancel{12} & & +12 \\ \hline n & = & 9 \end{array}$$

The number is 9.

Example 2

The second of two numbers is seven times the first. The sum of the two numbers is 32. Find the numbers.

Solution

Step 1: Write statements for the unknowns. Use only one variable.

Let n be the first number.
Let $7n$ be the second number.

The second of two numbers is seven times the first.

Step 2: Write an equation.

$$n + 7n = 32$$

The sum of the two numbers is 32.

Step 3: Solve the equation.

$$n + 7n = 32$$
$$8n = 32$$
$$\frac{8n}{8} = \frac{32}{8}$$
$$n = 4$$

The first number is 4.

Step 4: Find the second number if the first number is 4.

$$7n$$
$$= 7 \times 4$$
$$= 28$$

The second number is 28.

So, the numbers are 4 and 28.

Example 3

Yvonne has 20 more nickels than dimes in her piggy bank. If the total value of the nickels and dimes is \$8.50, how many coins of each type does she have?

Solution

Step 1: Write statements for the unknowns. Use only one variable.

Let the number of dimes be n.

The value of the nickels, the number of dimes, and the value of the dimes must also be represented by algebraic expressions.

A table can be used to organize the information.

Type of Coin	Number of Coins	Value in Cents
Dimes	n	$10n$
Nickels	$n+20$	$5(n+20)$
Total		850

The value of the dimes is 10 times the number of dimes.

The value of the nickels is 5 times the number of nickels.

Yvonne has 20 more nickels than dimes.

This is given.

Step 2: Write an equation.

$$5(n+20)+10n=850$$

Step 3: Solve the equation.

$$\begin{aligned}
5(n+20)+10n &= 850\\
5n+100+10n &= 850\\
5n+10n+100 &= 850\\
15n+\cancel{100} &= 850\\
-\cancel{100} \quad &\ \ -100\\
15n &= 750\\
\frac{15n}{15} &= \frac{750}{15}\\
n &= 50
\end{aligned}$$

Yvonne has 50 dimes in her piggy bank.

Step 4: Find the number of nickels if the number of dimes is 50.

$$\begin{aligned}
&n+20\\
&=50+20\\
&=70
\end{aligned}$$

Yvonne has 70 nickels in her piggy bank.

So, Yvonne has 50 dimes and 70 nickels in her piggy bank.

Glossary

Integers: the set of numbers that consists of the whole numbers and their opposites

$I = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Natural numbers: the set of numbers that consists of the counting numbers

$N = \{1, 2, 3, \ldots\}$

Perfect square: a number that has a natural number as its principal square root

64 is a perfect square because $\sqrt{64} = 8$.

Principal square root: the positive number which, when multiplied by itself, gives the original number

The principal square root of 36 is 6.

Rational numbers: the set of numbers that consists of numbers that can be written as a fraction of the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$

Repeating decimal: all decimal numbers that have digits that repeat singly or in a block of two or more

$0.5 = 0.5\bar{0}$ $\qquad$ $0.716\,161\,6\ldots = 0.7\overline{16}$

Square roots: numbers which, when multiplied by themselves, give the original number

4 and -4 are the square roots of 16, since $4 \times 4 = (-4) \times (-4) = 16$.

Terminating decimals: decimal numbers that do not repeat except for a zero on the end

0.5 $\qquad$ 1.8 $\qquad$ 237.164

Whole numbers: the set of numbers that consists of the natural numbers and zero

$W = \{0, 1, 2, 3, \ldots\}$

Suggested Answers

Section 1: Activity 1

1. Rounding is a better method of estimating in this case since it gives an answer that is closer to the actual answer.

2. You can use fewer keystrokes by changing it to a subtraction question.

   1 1 0 • 9 − 8 5 • 9 =

   This method uses one less keystroke.

3. You can eliminate one keystroke before using a calculator by changing the question from subtraction to addition mentally.

5 8 • 8 + 7 • 2 =

4. a. **Estimate**

$$\begin{aligned} -19.8+(-26.4) &\doteq -20+(-26) \\ &\doteq -(20+26) \\ &\doteq -46 \end{aligned}$$

Calculator

1 9 • 8 +/-
+ 2 6 • 4 +/- =

-46.2

b. **Estimate**

$$\begin{aligned} 43.7-(-21.4) &\doteq 44-(-21) \\ &\doteq 44+21 \\ &\doteq 65 \end{aligned}$$

Calculator

4 3 • 7
− 2 1 • 4 +/- =

65.1

c. **Estimate**

$$\begin{aligned} 148.5-216.8 &\doteq 149-217 \\ &\doteq -(217-149) \\ &\doteq -68 \end{aligned}$$

Calculator Method

1 4 8 • 5
− 2 1 6 • 8 =

-68.3

d. **Estimate**

$$\begin{aligned} 87.6+(-194.2) &\doteq 88-194 \\ &\doteq -(194-88) \\ &\doteq -106 \end{aligned}$$

Calculator

8 7 • 6

+ 1 9 4 • 2 +/− =

−106.6

5. Estimate Loedemal's time.

$$1 \text{ min } 27.38 \text{ s} + 0.28 \text{ s} \doteq 1 \text{ min } 27.4 \text{ s} + 0.3 \text{ s}$$
$$\doteq 1 \text{ min } 27.7 \text{ s}$$

$$\frac{28}{100} = 0.28$$

Enter the keystrokes as follows to find Loedemal's actual time.
Note: Enter only the seconds into the calculator; then add one minute to the final answer.

2 7 • 3 8 + • 2 8 =

27.66

Therefore, Loedemal's actual time was 1 min 27.66 s.

6. Estimate the profit or the loss.

$$-7.8 + (-2.7) + 5.6 + (-1.2) + 11.9 \doteq -8 + (-3) + 6 + (-1) + 12$$
$$\doteq 6$$

Enter the keystrokes as follows:

7 • 8 +/− + 2 • 7 +/−

+ 5 • 6 + 1 • 2 +/−

+ 1 1 • 9 =

5.8

The company had an overall profit of 5.8% over the five-year period.

7. Answers will vary. The following is a sample answer.

7 • 8 + 2 • 7 + 1 • 2 =

+/− + 5 • 6 + 1 1 • 9 =

5.8

This method uses one less keystroke.

8. Answers will vary. The use of the calculator was more efficient by mentally keeping track of which numbers are positive and which are negative.

9. **a.** Estimate the closing price by rounding each day's change to the nearest twenty-five cents.

$$16.25 + 0.15 + 0.35 - 0.25 - 0.10 + 0.50$$
$$\doteq 16.25 + 0.25 + 0.25 - 0.25 - 0.00 + 0.50$$
$$\doteq 17.00$$

Enter the keystrokes as follows:

1 6 . 2 5 + . 1 5
+ . 3 5 − . 2 5
− . 1 + . 5 =

16.9

The closing price of Telus stock on Friday was $16.90.

b. Yes, the calculated price is close to the estimated price. The estimate shows whether or not the decimal is in the correct place. For example, if you obtained an answer of $169.00 or $1.69 as the closing price on your calculator, then the estimate will show you that something is wrong. An incorrect decimal place in the entry of one of the daily price changes is an error that is hard to detect without an estimate.

10. **a.** **Estimate**

$$-3.81 \times 5.2 \doteq -4 \times 5$$
$$\doteq -20$$

Calculator

3 . 8 1 +/−
× 5 . 2 =

-19.812

b. **Estimate**

$$84.6 \div 9 \doteq 85 \div 9$$
$$\doteq 9$$

Calculator

8 4 . 6 ÷ 9 =

9.4

c. **Estimate**

$$20 \div (-3.5) \doteq 20 \div (-4)$$
$$\doteq -5$$

Calculator

2 0 ÷ 3 • 5 +/– =

-5.714285714

d. **Estimate**

$$-15 \times (-7.1) \doteq -15 \times (-7)$$
$$\doteq 105$$

Calculator

1 5 +/– × 7 • 1 +/– =

106.5

11.

Fraction	Decimal	Percent
$\frac{3}{8}$	0.375	37.5%
$\frac{1}{6}$	$0.1\overline{6}$	$16\frac{2}{3}\%$
$\frac{7}{8}$	0.875	87.5%
$\frac{5}{4}$ or $1\frac{1}{4}$	1.25	125%
$\frac{5}{6}$	$0.8\overline{3}$	$83\frac{1}{3}\%$
$\frac{5}{9}$	$0.\overline{5}$	$55\frac{5}{9}\%$
$\frac{11}{10}$ or $1\frac{1}{10}$	1.1	110%

12. **Estimate**

$$2\,500\,000 \div 8.5 \doteq 3\,000\,000 \div 10$$
$$\doteq 300\,000$$

Calculator

2 5 0 0 0 0 0 ÷ 8 • 5 =

294 117.6471

It took 294 118 truckloads.

13. a. **Estimate**

$$2295 \times 15\% \doteq 2000 \times 0.15$$
$$\doteq 300$$

Calculator

2 2 9 5 × . 1 5 =

344.25

They saved $344.25.

b. **Estimate**

$$2295 - 344.25 \doteq 2300 - 300$$
$$\doteq 2000$$

Calculator

2 2 9 5
− 3 4 4 . 2 5 =

1950.75

The sale price was $1950.75.

c. **Estimate**

$$1950.75 \times 0.07 \doteq 2000 \times 0.10$$
$$\doteq 200$$

$$2000 + 200 \doteq 2200$$

Calculator

1 9 5 0 . 7 5
× . 0 7 =

136.55

The GST was $136.55.

1 9 5 0 . 7 5
+ 1 3 6 . 5 5 =

2087.30

The total amount they paid was $2087.30.

14. a. **Estimate**

According to the graph, the car travels 21 m in $\frac{3}{4}$ s at 100 km/h.

1 s is $\frac{1}{4}$ or 0.25 more than $\frac{3}{4}$ s. Therefore, 1 s is $\frac{0.25}{0.75}$ or $\frac{1}{3}$ larger than $\frac{3}{4}$ s.

$$\frac{1}{3} \times 21 = 7$$

The car will travel about 7 m more or a total of about $21 \text{ m} + 7 \text{ m} = 28 \text{ m}$ in 1 s.

Calculator

Use a ratio.

$$\frac{1}{0.75} = \frac{x}{21}$$
$$0.75x = 1 \times 21$$
$$0.75x = 21 \quad \longleftarrow \text{Divide each side by 0.75.}$$
$$x = \frac{21}{0.75}$$

Use the following keystrokes.

2 1 ÷ • 7 5 =

28.

The car will travel 28 m in 1 s.

b. The car will travel 28 m during the driver's perception time, 21 m during the reaction time, and 66 m during the braking time. Total distance travelled is the sum of these three distances.

Estimate

$$28 + 21 + 66 \doteq 30 + 20 + 70$$
$$\doteq 120 \text{ m}$$

Calculator

2 8 + 2 1 + 6 6 =

115.

The total distance the car will travel is 115 m.

c. **Estimate**

$$115 \div 6 \doteq 100 \div 5$$
$$\doteq 20$$

Calculator

1 1 5 ÷ 6 =

19.1666667

The distance is equal to about 19 car lengths.

d. **Estimate**

$$100 \div 15 \doteq 100 \div 20$$
$$\doteq 5$$

Calculator

1 0 0 ÷ 1 5 =

6.6666666 7

The driver should stay approximately 7 car lengths behind a vehicle while both are travelling at 100 km/h.

e. Since both cars are travelling at 100 km/h, each car needs the same stopping distance. The distance the driver must stay behind is to allow for perception time and reaction time.

15. Answers will vary. A sample chart is given.

	A	B	C	D
1	Item	Store 1 Cost	Store 2 Cost	Store 3 Cost
2				
3	peas	1.09	1.29	1.15
4	chicken soup	1.25	0.99	1.05
5	hamburger	5.29	4.99	4.79
6				
7	Total	7.63	7.27	6.99

Store 3 gives the best value.

16. **a.** The change in value is read from the Net Change column. Therefore, the change in value is $+0.10$.

b. $1200 \times 0.10 = 120$

The change in value of all of Ria's shares is $120.

c. $1200 \times 12.25 = 14\ 700$

The total value of Ria's shares at the end of the day is $14 700.

17. It is 1:20 A.M. in Montreal when the plane leaves Edmonton. After a 4.5 h flight, it would be 5:50 A.M. in Montreal.

18. **a.**
$$\begin{aligned} s &= \frac{d}{t} \\ &= \frac{2976}{4.5} \\ &\doteq 661.333\ 333\ 3 \end{aligned}$$

The average speed of the plane is about 661 km/h.

b. difference = cruising speed – average speed

$$\begin{aligned} &\doteq 830 \text{ km/h} - 661 \text{ km/h} \\ &\doteq 169 \text{ km/h} \end{aligned}$$

The following are possible reasons for the difference between the average flight speed and the cruising speed of the aircraft.

- average speed includes landing and takeoff
- winds will affect the average speed
- size of payload affects the average speed

20. **a.** $4.5 \text{ h} = 4.5 \cancel{\text{h}} \times 60 \text{ min}/\cancel{\text{h}}$

$$= 270 \text{ min}$$

$$\begin{aligned} \text{fuel consumed} &= 90 \times 270 \\ &= 24\,300 \end{aligned}$$

The flight to Montreal consumed 24 300 kg of fuel.

b. The fuel required can be calculated using a ratio of the distance from Edmonton to Montreal to the distance from Montreal to Miami.

$$\begin{aligned} \frac{2976}{1413} &= \frac{24\,300}{x} \\ 2976x &= 24\,300 \times 1413 \\ x &= \frac{24\,300 \times 1413}{2976} \\ &\doteq 11\,537.600\,81 \end{aligned}$$

The flight from Montreal to Miami would require approximately 11 538 kg of fuel.

The plane has $34\,200 - 24\,300 = 9900$ kg of fuel left. Therefore, the plane would have to be refueled.

20. **a.** Substitute into the formula and calculate.

$$\begin{aligned} {}^\circ\text{C} &= \frac{5}{9}({}^\circ\text{F} - 32) \\ &= \frac{5}{9}(93 - 32) \\ &= \frac{5}{9}(61) \\ &\doteq 33.888\,888\,89 \end{aligned}$$

The temperature was about 34°C.

b. **Method 1**

5 ÷ 9 = × (9 3 – 3 2) =

Method 2

9 3 – 3 2 = × 5 ÷ 9 =

21. $$\begin{aligned}\text{Hamel's earnings} &= (4.5+5+2.5+5.5+7.5+8.5)\times 7.00\\ &= 33.5\times 7.00\\ &= 234.50\end{aligned}$$

Hamel earned \$234.50 that week.

$$\begin{aligned}\text{Annette's earnings} &= (5.5+3+6.5+3.5+4.5+7.5)\times 7.50\\ &= 30.5\times 7.50\\ &= 228.75\end{aligned}$$

Annette earned \$228.75 that week.

22. Yes, since length can only be thought of as a positive value.

23. **a.** $\sqrt{100}=10$ **b.** $\sqrt{256}=16$

c. $-\sqrt{81}=-9$ **d.** $-\sqrt{400}=-20$

24. **Estimate**

$$\begin{aligned}A &= \pi r^2\\ 153.9 &= \pi r^2\\ 150 &\doteq 3r^2\\ \frac{150}{3} &\doteq r^2\\ r^2 &\doteq 50\\ r &\doteq \sqrt{50}\\ &\doteq 7\end{aligned}$$

Calculator

$$\begin{aligned}A &= \pi r^2\\ 153.9 &= \pi r^2\\ \frac{153.9}{\pi} &= r^2\\ r &= \sqrt{\frac{153.9}{\pi}}\end{aligned}$$

1 5 3 • 9 ÷ π = $\sqrt{\ }$ =

6.999 135053

The radius of the circle is about 7 cm.

25. **a.** Only the positive value of the radius was used because radius is a length and cannot be negative.

b. The name given this value is the principal square root.

c. No, a calculator only gives the positive value for the square root of a positive number.

d. The calculator gives an error message when you try to take the square root of a negative number. This is because there is no number that, when multiplied by itself, gives a negative number.

26. **a.** $b^2=c^2-a^2$

b. $b^2 = c^2 - a^2$

$b^2 = 13^2 - 5^2$

$b^2 = 169 - 25$

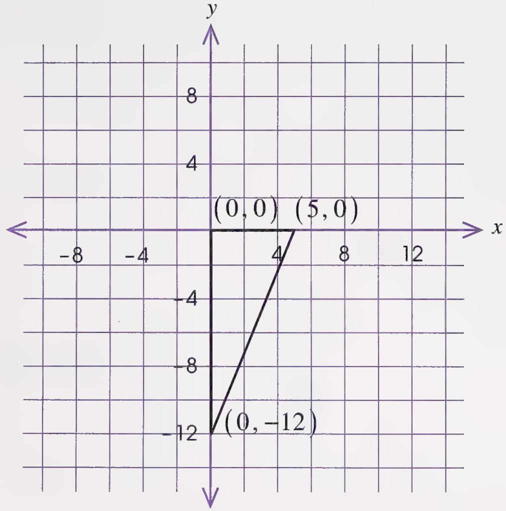
$b^2 = 144$

$b = \sqrt{144}$

$= 12$

The length of side b is 12 cm. You only use the positive (principal) square root since length can only be positive.

c.



The coordinates of the other two vertices are $(5, 0)$ and $(0, -12)$.

d. Yes, both positive and negative numbers are used to name the coordinates.

No, these numbers are not lengths. They are positions (or locations) on the coordinate plane.

27. a. Estimate

$\sqrt{16} < \sqrt{20} < \sqrt{25}$

$4 < \sqrt{20} < 5$

$\therefore \sqrt{20} \doteq 4.5$

Calculator

√ 2 0 =

4.472 135955

$\therefore \sqrt{20} \doteq 4.47$

b. Estimate

$\sqrt{4} < \sqrt{7} < \sqrt{9}$

$2 < \sqrt{7} < 3$

$\therefore \sqrt{7} \doteq 2.6$

Calculator

√ 7 =

2.6457513111

$\therefore \sqrt{7} \doteq 2.65$

c. **Estimate**

$$\sqrt{900} < \sqrt{1215} < \sqrt{1600}$$
$$30 < \sqrt{1215} < 40$$
$$\therefore \sqrt{1215} \doteq 35$$

Calculator

[√] [1] [2] [1] [5] [=]

34.856850 12

$$\therefore \sqrt{1215} \doteq 34.86$$

d. **Estimate**

$$-\sqrt{36} < -\sqrt{32} < -\sqrt{25}$$
$$-6 < -\sqrt{32} < -5$$
$$-\sqrt{32} \doteq -5.6$$

Calculator

[√] [3] [2] [=] [+/−]

-5.656854249

$$\therefore -\sqrt{32} \doteq -5.66$$

e. **Estimate**

$$\sqrt{9} < \sqrt{15} < \sqrt{16}$$
$$3 < \sqrt{15} < 4$$
$$\therefore \sqrt{15} \doteq 3.9$$

Calculator

[√] [1] [5] [=]

3.872983346

$$\therefore \sqrt{15} \doteq 3.87$$

f. **Estimate**

$$\sqrt{625} < \sqrt{873} < \sqrt{900}$$
$$25 < \sqrt{873} < 30$$
$$\therefore \sqrt{873} \doteq 29$$

Calculator

[√] [8] [7] [3] [=]

29.54657341

$$\therefore \sqrt{873} \doteq 29.55$$

28. **a.** The digits filled the entire display for every answer.

b. Yes, there would be as many digits as possible displayed.

c. These numbers are called non-terminating, non-repeating decimals.

29. First, estimate the answer.

area of cross stitch picture

$$\begin{aligned} A &= s^2 \\ &= 24^2 \\ &\doteq 25^2 \\ &\doteq 625 \end{aligned}$$

area of drymount board

$$\begin{aligned} A &\doteq 2 \times 625 \\ &\doteq 1250 \end{aligned}$$

length of side of drymount board

$$\begin{aligned} s &= \sqrt{A} \\ &= \sqrt{1250} \\ &\doteq 35 \end{aligned}$$

$$\sqrt{900} < \sqrt{1250} < \sqrt{1600}$$
$$30 < \sqrt{1250} < 40$$

Now calculate the actual answer.

area of cross stitch picture

$$\begin{aligned} A &= s^2 \\ &= 24^2 \\ &= 576 \end{aligned}$$

area of drymount board

$$\begin{aligned} A &= 2 \times 576 \\ &= 1152 \end{aligned}$$

length of side of drymount board

$$\begin{aligned} s &= \sqrt{A} \\ &= \sqrt{1152} \\ &\doteq 33.941\,125\,5 \end{aligned}$$

The length of a side of the drymount board is approximately 34 cm.

30. Find the area of the pasture.

$$\begin{aligned} A &= 4 \times 10\,000 \\ &= 40\,000 \end{aligned}$$

The area of the pasture is $40\,000 \text{ m}^2$.

Find the length of each side of the pasture.

$$\begin{aligned} \ell &= \sqrt{A} \\ &= \sqrt{40\,000} \\ &= 200 \end{aligned}$$

The length of each side is 200 m.

Therefore, the pasture requires $200 \times 4 = 800$ m of fencing.

31. $$\begin{aligned} s &= \sqrt{150b} \\ &= \sqrt{150(28)} \\ &= \sqrt{4200} \\ &\doteq 64.807\,406\,98 \end{aligned}$$

According to the formula, Gina was travelling at approximately 65 km/h at the time of the accident. The police officer gave her the ticket because she was 15 km/h over the speed limit.

32. You can find the length of the side of the square by taking the square root of the area. Since the perimeter is four times the length of one side, multiply the length by 4 in order to calculate the perimeter.

Section 1: Activity 2

1. All operations within brackets are to be done first. Next, all operations involving exponents are to be completed. Then, the operations involving division and multiplication (in the order in which they appear) are to be done. Finally, the operations of addition and subtraction (in the order in which they appear) are to be completed.

2. a. $\boxed{\cdot}\ \boxed{5}\ \boxed{-}\ \boxed{\cdot}\ \boxed{7}\ \boxed{\times}\ \boxed{\cdot}\ \boxed{3}\ \boxed{=}$

$\boxed{0.29}$

b. $\boxed{(}\ \boxed{\cdot}\ \boxed{5}\ \boxed{-}\ \boxed{\cdot}\ \boxed{7}\ \boxed{)}$

$\boxed{-0.2}$

$\boxed{\times}\ \boxed{\cdot}\ \boxed{3}\ \boxed{=}$

$\boxed{-0.06}$

3. The calculator automatically did the multiplication first.

4. The brackets made the calculator do the enclosed operation first.

5. No, the answers to questions 2.a. and 2.b. are different because a different operation was performed first.

6. a. **Estimate**

$$5.27 + 9.3 \times 2.1 \doteq 5 + 9 \times 2 \quad \leftarrow \text{Multiply.}$$
$$\doteq 5 + 18 \quad \leftarrow \text{Add.}$$
$$\doteq 23$$

Calculator

5 • 2 7 + 9 • 3 × 2 • 1 =

24.8

b. **Estimate**

$$-3.98 \times 1.4 \div 0.7 + 7.56 \doteq -4 \times 1 \div 1 + 8 \quad \leftarrow \text{Multiply.}$$
$$\doteq -4 \div 1 + 8 \quad \leftarrow \text{Divide.}$$
$$\doteq -4 + 8 \quad \leftarrow \text{Add.}$$
$$\doteq 4$$

Calculator

3 • 9 8 +/₋ × 1 • 4 ÷ • 7 + 7 • 5 6 =

-0.4

Note: This answer is still considered to be reasonable even though it does not have the same sign as the estimated answer.

c. **Estimate**

$$(3.34 - 7.21)4 - 3.50 \doteq (3 - 7)4 - 4 \quad \leftarrow \text{Work in brackets first; subtract.}$$
$$\doteq (-4)4 - 4 \quad \leftarrow \text{Multiply.}$$
$$\doteq -16 - 4 \quad \leftarrow \text{Subtract.}$$
$$\doteq -20$$

Calculator

(3 • 3 4 − 7 • 2 1) × 4 − 3 • 5 =

- 18.98

d. **Estimate**

$$\frac{5.3+25.1-19.3}{4} \doteq \frac{5+25-19}{4} \quad \longleftarrow \text{Add.}$$
$$\doteq \frac{30-19}{4} \quad \longleftarrow \text{Subtract.}$$
$$\doteq \frac{11}{4} \quad \longleftarrow \text{Divide.}$$
$$\doteq 3$$

Calculator

(5 • 3 + 2 5 • 1
− 1 9 • 3) ÷ 4 =

2.775

7. The brackets were used because the entire numerator is divided by 4.

8. Instead of brackets, you could use the equal key before pressing the division key. This method may or may not be a better way, but it does save one keystroke.

9. **a.** $2\times 11.99+7.99 \doteq 2\times 12+8$ ⟵ Multiply.
$\doteq 24+8$ ⟵ Add.
$\doteq 32$

The estimate is \$32.

b. $3\times 2.89+2\times 34.98 \doteq 3\times 3+2\times 35$ ⟵ Multiply.
$\doteq 9+70$ ⟵ Add.
$\doteq 79$

The estimate is \$79.

c. $6\div 3\times 1.89 \doteq 6\div 3\times 2$ ⟵ Divide.
$\doteq 2\times 2$ ⟵ Multiply.
$\doteq 4$

The estimate is \$4.

10. Both taxes are calculated on the cost of the bicycle as follows:

Estimate.

$$\begin{aligned} & 375+(375\times 7\%)+(375\times 5\%) \\ &\doteq 400+(400\times 10\%)+(400\times 10\%) \\ &\doteq 400+(400\times 0.10)+(400\times 0.10) \\ &\doteq 400+40+40 \\ &\doteq 480 \end{aligned}$$

Enter the keystrokes as follows:

3 7 5 + 3 7 5 × • 0 7
+ 3 7 5 × • 0 5 =

420.

You can also enter the keystrokes as follows:

3 7 5 + 3 7 5
× • 1 2 =

420.

Samantha had to pay $420 for her bicycle.

11. **a.** $$\begin{aligned} 135\,000+(135\,000\times 7\%) &= 135\,000+(135\,000\times 0.07) \\ &= 135\,000+9450 \\ &= 144\,450 \end{aligned}$$

The Van Dusen family paid $144 450 including GST.

b. $$\begin{aligned} 135\,000+(135\,000\times 5\%) &= 135\,000+(135\,000\times 0.05) \\ &= 135\,000+6750 \\ &= 141\,750 \end{aligned}$$

The O'Rourkes paid $141 750 for the house.

c. $$\begin{aligned} 141\,750-(141\,750\times 5\%) &= 141\,750-(141\,750\times 0.05) \\ &= 141\,750-7087.50 \\ &= 134\,662.50 \end{aligned}$$

The O'Rourkes sold the house for $134 662.50.

12. $$\begin{aligned} v &= 0.25q+0.1d \\ &= 0.25(31)+0.1(82) \\ &= 7.75+8.2 \\ &= 15.95 \end{aligned}$$

The face value of Matthew's collection is $15.95.

13. $t = \sqrt{\dfrac{h}{4.9}}$

$= \sqrt{\dfrac{39.7}{4.9}}$

The height of the fall is $41.8 - 2.1 = 39.7$.

Estimate.

$\sqrt{\dfrac{39.7}{4.9}} \doteq \sqrt{\dfrac{40}{5}}$

$\doteq \sqrt{8}$

$\doteq 2.8$

$\sqrt{4} < \sqrt{8} < \sqrt{9}$

$2 < \sqrt{8} < 3$

Enter the keystrokes as follows:

3 9 • 7 ÷ 4 • 9 =

8.102040816

√ =

2.846408406

It would take the stone approximately 2.8 s to reach the water.

14. $t = \sqrt{\dfrac{h}{4.9}}$

$= \sqrt{\dfrac{24.1}{4.9}}$

Estimate.

$\sqrt{\dfrac{24.1}{4.9}} \doteq \sqrt{\dfrac{25}{5}}$

$\doteq \sqrt{5}$

$\doteq 2.2$

$\sqrt{4} < \sqrt{5} < \sqrt{9}$

$2 < \sqrt{5} < 3$

Enter the following keystrokes.

2 4 • 1 ÷ 4 • 9 =

4.918367347

√ =

2.217739242

It would take approximately 2.2 s for the drop of water to reach the ground.

15. $t = \sqrt{\dfrac{h}{4.9}}$

$= \sqrt{\dfrac{54.5}{4.9}}$

Estimate.

$\sqrt{\dfrac{54.5}{4.9}} \doteq \sqrt{\dfrac{55}{5}}$

$\doteq \sqrt{11}$

$\doteq 3.3$

$\sqrt{9} < \sqrt{11} < \sqrt{16}$

$3 < \sqrt{11} < 4$

Enter the following keystrokes.

5 4 • 5 ÷ 4 • 9 =

11.12244898

√ =

3.33503358

It took the two objects about 3.3 s to reach the ground.

Now Try This

16. a. √ 4 =

2.

b. √ 4 0 =

6.32455532

c. √ 4 0 0 =

20.

d. √ 4 0 0 0 =

63.2455532

e. √ 4 0 0 0 0 =

200.

f. $\boxed{\sqrt{\ }}$ $\boxed{4}$ $\boxed{0}$ $\boxed{0}$ $\boxed{0}$ $\boxed{0}$ $\boxed{0}$ $\boxed{=}$

632.455532

17. The numbers can be put into two groups of three. Within each group, each number is 100 times the previous one, and each square root is $\sqrt{100}$ or 10 times the previous answer.

Group 1

$\sqrt{4} = 2$

Since $400 = 100 \times 4$, then $\sqrt{400} = \sqrt{100} \times \sqrt{4}$
$= 10 \times 2$
$= 20$

Since $40\,000 = 100 \times 400$, then $\sqrt{40\,000} = \sqrt{100} \times \sqrt{400}$
$= 10 \times 20$
$= 200$

Group 2

$\sqrt{40} \doteq 6.324\,555\,32$

Since $4000 = 100 \times 40$, then $\sqrt{4000} \doteq \sqrt{100} \times \sqrt{40}$
$\doteq 10 \times 6.324\,555\,32$
$\doteq 63.245\,553\,2$

Since $400\,000 = 100 \times 4000$, then
$\sqrt{400\,000} \doteq \sqrt{100} \times \sqrt{4000}$
$\doteq 10 \times 63.245\,553\,2$
$\doteq 632.455\,532$

18. a. If $\sqrt{16} = 4$, then $\sqrt{160\,000} =$ 400 .

b. If $\sqrt{39} \doteq 6.245$, then $\sqrt{3900} \doteq$ 62.45 .

c. If $\sqrt{225} = 15$, then $\sqrt{2.25} =$ 1.5 .

d. If $\sqrt{7.5} \doteq 2.7386$, then $\sqrt{750} \doteq$ 27.386 .

e. If $\sqrt{25} = 5$, then $\sqrt{25\,000\,000} =$ 5000 .

Section 1: Activity 3

1. $\boxed{+}$ instructs the calculator to add.

$\boxed{\times}$ instructs the calculator to multiply.

$\boxed{=}$ displays the sum of 12.7 and the product of 2 and 1.3.

$\boxed{\times}$ instructs the calculator to multiply.

$\boxed{(}$ instructs the calculator to perform the previous operation on the result within the brackets.

$\boxed{+}$ instructs the calculator to add.

× instructs the calculator to multiply.

) displays the result of what is within brackets.

= displays the result of multiplication.

− instructs the calculator to subtract.

× instructs the calculator to multiply.

= displays the result of multiplication and subtraction.

2. × instructs the calculator to multiply.

= displays the product of 12.7 and 8.5.

Min stores the product of 12.7 and 8.5.

+ instructs the calculator to add.

× instructs the calculator to multiply.

= displays the sum of 12.7 and the produce of 2 and 1.3.

× instructs the calculator to multiply.

(instructs the calculator to perform the previous operation on the result within the brackets.

+ instructs the calculator to add.

× instructs the calculator to multiply.

) displays the result within brackets.

= displays the result of multiplication.

− instructs the calculator to subtract.

MR retrieves the previously stored value.

= displays the result of previous subtraction.

3. + instructs the calculator to add.

× instructs the calculator to multiply.

= displays the sum of 8.5 and the product of 2 and 1.3.

Min stores the previous sum.

+ instructs the calculator to add.

× instructs the calculator to multiply.

= displays the sum of 12.7 and the product of 2 and 1.3.

× instructs the calculator to multiply.

MR retrieves the previously stored value.

− instructs the calculator to subtract and displays the product of the sum and the stored value.

× instructs the calculator to multiply.

= displays the result of previous subtraction.

4. **Method 1**

12 operator + 22 number entry = 34 keystrokes

Method 2

15 operator + 22 number entry = 37 keystrokes

Method 3

12 operator + 22 number entry = 34 keystrokes

5. Method 2 is least efficient since it uses the most keystrokes.

6. Method 1 is done in the order the question was written and uses brackets. Method 3 uses the memory storage and memory recall keys.

7. Answers will vary. You may prefer Method 1 since it is done in the order the question was written. However, using the memory storage keys have an advantage in multi-step questions in that you can use this stored value several times if necessary.

8. **a.** The keystroke sequence in B is the most efficient since it gives the correct solution and has the fewest number of keystrokes.

b. Using rules of BEDMAS the original question is asking for the product of 3.2 and 5.3 to be subtracted from 6.1. Keystroke sequence D is finding the difference of 6.1 and 3.2, then multiplying the result by 5.3. Keystroke sequence E is multiplying 3.2 and 5.3, then subtracting 6.1 from the product.

9. Enter 2.6 and 19.3; find the sum; and place the sum in memory. Then, enter 28.2 and 18.1; find the sum; and divide by the sum in memory.

10. **a.** (2 8 • 2 + 1 8 • 1
) ÷ (2 • 6 +
1 9 • 3) =

b. You can use the equal key in front of the division key instead of the first two brackets.

11. **Estimate**

$$\frac{24.6+31.7}{8.8+4.3-7.1} \doteq \frac{25+32}{9+4-7}$$
$$\doteq \frac{57}{6}$$
$$\doteq 10$$

Calculator

Method 1

8 • 8 + 4 • 3
− 7 • 1 = Min 2 4 • 6
+ 3 1 • 7 = ÷ MR =

9.383333333

Method 2

2 4 . 6 + 3 1 . 7 =

÷ (8 . 8 + 4 . 3

− 7 . 1) =

9.383333333

The value of the expression is 9.4, rounded to one decimal.

12. Answers will vary. You may have used one different method. However, it may have been less efficient.

13. The reciprocal function changed the divisor (6) to $\frac{1}{6}$ in decimal form. Yes the answer to the expression is the same.

14. The division operation was changed to a multiplication operation.

15. No, this method is not more efficient. This method used the same number of keystrokes as Method 1 and one more keystroke than Method 2.

16. **Estimate**

Outer Border

$$\begin{aligned}
A &= A \text{ of carpet} - A \text{ of inner square and narrow strip} \\
&= (6.6\times 7.9) - (3.4+0.3+0.3)\times(4.7+0.3+0.3) \\
&\doteq 7\times 8 - (3+0.5+0.5)\times(5+0.5+0.5) \\
&\doteq 56-(4)\times(6) \\
&\doteq 56-24 \\
&\doteq 32
\end{aligned}$$

The estimated area is 32 m.

Narrow Strip

$$\begin{aligned}
A &= A \text{ of inner square and narrow strip} - A \text{ of inner square} \\
&= (3.4+0.3+0.3)\times(4.7+0.3+0.3) - (3.4\times 4.7) \\
&\doteq (3+0.5+0.5)\times(5+0.5+0.5) - (3\times 5) \\
&\doteq (4)\times(6)-15 \\
&\doteq 24-15 \\
&\doteq 9
\end{aligned}$$

The estimated area is 9 m.

Inner Rectangle

$$\begin{aligned} A &= 3.4 \times 4.7 \\ &\doteq 3 \times 5 \\ &\doteq 15 \end{aligned}$$

The estimated area is 15 m.

Calculator

Outer Border

(3 • 4 + • 3
+ • 3) × (4 • 7
+ • 3 + • 3) = Min

21.2

6 • 6 × 7 • 9 =
− MR =

30.94

The area of the outer border of the carpet is 30.94 m^2.

Narrow Strip

3 • 4 × 4 • 7 = Min

15.98

3 • 4 + • 3
+ • 3 = × (4 • 7
+ • 3 + • 3) =

21.2

− MR =

5.22

The area of the narrow strip of the carpet is 5.22 m^2.

Inner Rectangle

3 • 4 × 4 • 7 =

15.98

The area of the inner rectangle of the carpet is 15.98 m^2.

17. **a.** 4 9 0 • 7 ÷ (π × 1 0 • 5) = $\sqrt{\ }$ =

3.8569005711

The radius of the can is about 3.9 cm.

b. π × 1 0 • 5 = $\frac{1}{x}$ × 4 9 0 • 7 = $\sqrt{\ }$ =

3.8569005711

The radius of the can is about 3.9 cm.

18. Both methods use the same number of keystrokes; thus, both methods would have the same efficiency.

Did You Know?

19. **a.** 83×87

$8 \times 9 = 72$
$3 \times 7 = 21$

Therefore, the product is 7221.

b. 94×96

$9 \times 10 = 90$
$4 \times 6 = 24$

Therefore, the product is 9024.

c. 75×75

$7 \times 8 = 56$
$5 \times 5 = 25$

Therefore, the product is 5625.

d. 31×39

$3 \times 4 = 12$
$1 \times 9 = 9$

Write 9 as 09.

Therefore, the product is 1209.

e. 42×48

$4 \times 5 = 20$
$2 \times 8 = 16$

Therefore, the product is 2016.

f. 53×57

$5 \times 6 = 30$
$3 \times 7 = 21$

Therefore, the product is 3021.

Section 1: Follow-up Activities

Extra Help

1.

Expression $\dfrac{204.41}{3.4\times(18.7-12.5)}$	
Keystrokes	**Description**
1 8 • 7 − 1 2 • 5 = × 3 • 4 = Min 2 0 4 • 4 1 ÷ MR =	Evaluate the denominator first using equal keys; then use the memory keys to solve the expression.
1 8 • 7 − 1 2 • 5 = × 3 • 4 = $\frac{1}{x}$ × 2 0 4 • 4 1 =	Evaluate the denominator first using equal keys; then multiply its reciprocal by the numerator.
2 0 4 • 4 1 ÷ (3 • 4 × (1 8 • 7 − 1 2 • 5)) =	Enter the expression in the order it appears using more than one set of brackets.
(3 • 4 × (1 8 • 7 − 1 2 • 5)) Min 2 0 4 • 4 1 ÷ MR =	Evaluate the denominator first using brackets; then use the memory keys to solve the expression.
(3 • 4 × (1 8 • 7 − 1 2 • 5)) $\frac{1}{x}$ × 2 0 4 • 4 1 =	Evaluate the denominator first using brackets; then multiply its reciprocal by the numerator.

2. Two methods have been shown in each question. You may have a different method, but your answer should be the same.

a. Method 1

3 • 1 − 4 • 5 =
× (6 • 2 + 1 • 4) =

- 10.64

Method 2

3 • 1 − 4 • 5 = Min
6 • 2 + 1 • 4 = × MR =

- 10.64

b. Method 1

9 • 3 4 × 5 • 7 8
÷ (8 • 7 4 +/−
+ 5 • 3 1) =

- 15.739 12536

Method 2

8 • 7 4 +/− + 5 • 3 1
= 1/x × (9 • 3 4
× 5 • 7 8) =

- 15.739 12536

c. Method 1

1 7 • 1 − 1 0 • 7
= × 9 • 8 = Min 8 • 7
− 5 • 9 + 3 • 1 =
× 3 • 1 ÷ MR =

0.29 16 1352

Method 2

1 7 • 1 − 1 0 • 7

= × 9 • 8 = $\frac{1}{x}$

Min 8 • 7 − 5 • 9

+ 3 • 1 = × 3 • 1 × MR =

0.29161352

Enrichment

1. Answers will vary. Perhaps you know or heard some good information about a particular company. You may know someone who works for or who owns some shares in the company.

2. Fluctuations may be caused by any of the following:

   - higher or lower company profits
   - a new discovery
   - overall market changes

3. Answers will vary. You may or may not believe that the stock market is a good place to invest money.

Section 2: Activity 1

1. **a.** Yes, there is a least natural number. It is 1.

b. There is no greatest natural number. You can always think of a greater natural number by adding 1 to it.

c. There are no natural numbers between 1 and 2, 2 and 3, 3 and 4, and so on.

2. **a.** The number 0 is a whole number but not a natural number.

b. There is a least whole number. It is 0. There is no greatest whole number.

c. There are no whole numbers between 0 and 1, 1 and 2, 2 and 3, and so on.

3. **a.** There is no least number and there is no greatest number on the integer number line.

b. There are no integers between -3 and -2, -2 and -1, -1 and 0, and so on.

4. There is no least number and there is no greatest number on the rational number line.

5. **a.** true **b.** false **c.** true

d. false **e.** false **f.** true

g. true **h.** true

6. **a.** $2 \div 9 = 0.222\ 222\ 222\ldots$

   b. $\sqrt{11} = 3.316\ 624\ 79\ldots$

   c. $\sqrt{89} = 9.433\ 981\ 132\ldots$

   d. $\sqrt{81} = 9$

   e. $\sqrt{50} = 7.071\ 067\ 812\ldots$

   f. $49 \div 6 = 8.166\ 666\ 666\ldots$

7. No, $\sqrt{11}$, $\sqrt{89}$, and $\sqrt{50}$ do not terminate or repeat.

8. The calculator has rounded the last digit in the answer to question 6.f. As a repeating decimal, the answer is written as $8.1\overline{6}$ or $8.1\dot{6}$.

9. From the digits on the calculator, it appears that the value of π does not repeat or terminate. The value of π is not a rational number.

10. The number $\frac{8}{0}$ is not a rational number because it has zero as a denominator, and you cannot divide by zero.

11. The number -8 is a rational number because it can be expressed as a ratio of two integers $\left(\frac{-8}{+1}\right)$. The number -8 is not a whole number because whole numbers do not include negative numbers.

12. Answers will vary. The following are some situations that may be represented by rational numbers.

• a growth of 3.5 cm	$+3.5$ cm
• a reduction of \$8.50	$-\$8.50$
• an increase of 10%	$+10\%$
• a decrease of $3\frac{1}{8}$	$-3\frac{1}{8}$

13. Other calculator methods are possible, but the answers should be the same.

   a. 2 ÷ 3 + • 7 =

   1.366666667

   The answer is 1.37, rounded to two decimal places.

   b. 3 + 1 ÷ 8 − 1 • 7 4 =

   1.385

   The answer is 1.39, rounded to two decimal places.

c. 2 + 1 ÷ 6 = Min
4 • 1 5 − MR =

1.983333333

The answer is 1.98, rounded to two decimal places.

14. The first tap can fill $\frac{1}{12}$ of the vat in 1 min.

The second tap can fill $\frac{1}{18}$ of the vat in 1 min.

Together, the two taps will fill $\frac{1}{12}+\frac{1}{18}$ of the vat in 1 min.

Use the following keystrokes to find the solution.

1 ÷ 1 2 + 1 ÷ 1 8 = $\frac{1}{x}$

7.2

The two taps will fill the vat in 7.2 min.

Section 2: Activity 2

1. **a.** Yes. Every integer can be written as a rational number.

b. No. Many rational numbers $\left(\text{like } \frac{1}{2},\ -0.7,\ \text{and } 3\frac{1}{6}\right)$ are not integers.

c. Yes. All whole numbers belong to the set of integers.

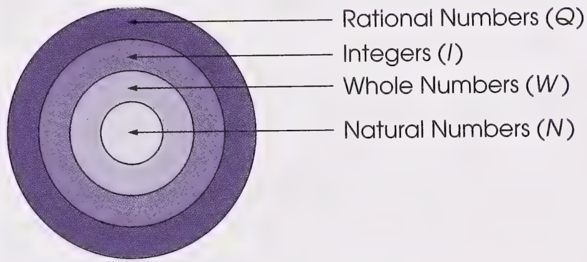
d. Yes. For every integer, you can find another integer (smaller or larger) by adding or subtracting 1.

e. No. Zero is neither positive or negative.

f. Yes. All integers are rational numbers.

g. 6, −2, 187, and so on

2. The answer is C.



Every natural number is a whole number since whole numbers include all natural numbers plus zero.

Every whole number is an integer because integers include whole numbers and their opposites.

Every integer is a rational number since all integers can be expressed as fractions $\left(-3=\frac{-3}{1},\ -2=\frac{-2}{1},\ -1=\frac{-1}{1},\ 0=\frac{0}{1}\right)$.

3. N: 5 — I: 0, 5, −4

W: 0, 5 — Q: 0, $-\frac{7}{8}$, 5, −4, 1.25

4. Since 5 is a counting number, it is also a natural number. All natural numbers are a part of the other sets of numbers; thus, 5 is also a whole number, an integer, and a rational number.

5. Answers will vary in a., c., and d.

   a. 0, 1, 2, 3, and so on

   All whole numbers are integers.

   b. The only whole number that is not a natural number is 0.

   c. $\frac{1}{2}$, −3.5, $-1\frac{5}{6}$, and so on

   d. 0, 1, 2, 3, and so on

   All whole numbers are rational numbers.

Section 2: Follow-up Activities

Extra Help

1.

Number	Type of Number			
	N	*W*	*I*	*Q*
−3			✔	✔
0		✔	✔	✔
$\frac{3}{5}$				✔
16	✔	✔	✔	✔
$\sqrt{25}$	✔	✔	✔	✔
$-\frac{11}{16}$				✔
$0.\overline{3}$				✔
$2\frac{2}{3}$				✔
−1.5				✔
1.404 004…				
−9			✔	✔
$\sqrt{5}$				

2. **a.** iv **b.** i **c.** iii **d.** ii

Enrichment

1. There are many more dots on the rational number line.

2. Yes. An example of a rational number between $-1\frac{1}{2}$ and −1 is $-1\frac{1}{3}$, and an example of a rational number between $\frac{1}{2}$ and 1 is $\frac{3}{4}$.

3. Yes, you can find more than one rational number between each of the pairs of numbers in question 2.

4. It indicates there are an infinite number of rational numbers.

5. Answers will vary.

   a. Change $\frac{3}{8}$ and $\frac{5}{12}$ to common denominators.

   $$\frac{3}{8}=\frac{9}{24} \qquad \frac{5}{12}=\frac{10}{24}$$

   There is no whole number between 9 and 10; so, you need to find equivalent fractions with a larger denominator.

   $$\frac{9}{24}=\frac{18}{48} \qquad \frac{10}{24}=\frac{20}{48}$$

   Thus, $\frac{19}{48}$ is between $\frac{18}{48}$ $\left(\text{or } \frac{3}{8}\right)$ and $\frac{20}{48}$ $\left(\text{or } \frac{5}{12}\right)$.

   $$\frac{18}{48}=\frac{36}{96} \qquad \frac{20}{48}=\frac{40}{96}$$

   You still must find another rational number; therefore, you must find equivalent fractions with larger denominators.

   Thus, $\frac{37}{96}$, $\frac{38}{96}$ $\left(\text{or } \frac{19}{48}\right)$, and $\frac{39}{96}$ are between $\frac{36}{96}$ $\left(\text{or } \frac{3}{8}\right)$ and $\frac{40}{96}$ $\left(\text{or } \frac{5}{12}\right)$.

   b. Write equivalent decimal numbers.

   $$1.6 = 1.60 \qquad 1.65$$

   Thus, 1.61, 1.62, 1.63, and 1.64 are between 1.6 and 1.65.

   c. $-0.4=-0.40 \qquad -0.3=-0.30$

   Thus, -0.39, $-0.38, \ldots$, -0.32, and -0.31 are between -0.4 and -0.3.

   d. $4.3 = 4.30 \qquad 4.\overline{3} = 4.33\ldots$

   Thus, 4.31 and 4.32 are between 4.3 and $4.\overline{3}$.

   e. $-\frac{5}{6}=-\frac{45}{54} \qquad -\frac{7}{9}=-\frac{42}{54}$

   Thus, $-\frac{44}{54}$ and $-\frac{43}{54}$ are between $-\frac{45}{54}$ $\left(\text{or } \frac{5}{6}\right)$ and $-\frac{42}{54}$ $\left(\text{or } -\frac{7}{5}\right)$.

   f. $\frac{5}{12} \qquad \frac{2}{3}=\frac{8}{12}$

   Thus, $\frac{6}{12}$ and $\frac{7}{12}$ are between $\frac{5}{12}$ and $\frac{8}{12}$ $\left(\text{or } \frac{2}{3}\right)$.

6. Add the two rational numbers; then divide by 2.

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